Modellunabhängige Suche nach Neutrinoquellen mit dem ANTARES Neutrinoteleskop

Model-independent search for neutrino sources with the ANTARES neutrino telescope

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Kurzzusammenfassung

Der Ursprung der hochenergetischen kosmischen Strahlung, die wir auf der Erde beobachten können, ist trotz jahrzehntelanger intensiver Forschung immer noch nicht eindeutig bekannt. Viele Theorien, die versuchen die Entstehung dieser Teilchen zu erklären, sagen auch einen Fluss von hochenergetischen Neutrinos aus dem Kosmos vorher. Zwar wurde vor kurzem die Existenz dieser Neutrinos bestätigt, die Quellen konnten dabei aber nicht identifiziert werden. Das ANTARES Neutrinoteleskop wurde im Mittelmeer 40 km von Toulon in einer Tiefe von 2475 m gebaut, um diese und andere Fragen zu klären. Es besteht aus 885 Photovervielfachern, die in einer dreidimensionalen Struktur angeordnet sind, um die Tscherenkovstrahlung zu detektieren, die von Teilchen ausgesendet wird, die bei der Interaktion von Neutrinos mit dem Meerwasser entstehen.

Die Interaktionen von Neutrinos in den ANTARES Daten zu identifizieren und zu rekonstruieren stellt herausfordernde Aufgaben dar. Die ersten beiden neuen Methoden, die in dieser Arbeit vorgestellt werden, versuchen diese Aufgaben möglichst effizient durch Mustererkennung zu lösen. Der erste Algorithmus klassifiziert die Daten in von unterhalb des Detektors kommende Neutrinosignale und von oberhalb kommende Teilchen, um einen Großteil der von oben kommenden atmosphärischen Muonen ausfiltern zu können. Der zweite Algorithmus verbessert die Richtungsrekonstruktion, indem er für jedes Neutrino den jeweils genausten Rekonstruktionsalgorithmus auswählt.

Der Hauptfokus dieser Arbeit liegt auf einer neuen Methodik, die die Verteilung der rekonstruierten Neutrinos auswertet. Im Gegensatz zu den meisten Analysen in ANTARES versucht diese Suche nicht, ein spezifisches, theoriebasiertes Modell einer Neutrinoquelle zu detektieren, sondern zielt darauf ab, die deutlichste Überfluktuation mit beliebiger Größe, Form und Verteilung der Neutrinos an einer beliebigen Position im Himmel zu finden. Um dieses Ziel zu erreichen wird die Dichte der Neutrinoereignisse in 180 voneinander unabhängigen Skalen ausgewertet. Wenn eine Region mit unerwartet hoher Neutrinodichte identifiziert worden ist, wird das potentielle Neutrinosignal mit Pseudoexperimenten mit zufälligen Neutrinoverteilungen verglichen, um seine Signifikanz zu bestimmen. Die Stärken einer derart flexiblen modellunabhängigen Suche liegen üblicherweise nicht in einer hohen Sensitivität bezüglich einer bestimmten Quellannahme, sondern im Entdecken einer Hypothese, die dann genauer ausgewertet werden kann.

In den Daten, die zwischen 2007 und 2012 aufgezeichnet wurden, wurde eine sehr große, ausgedehnte Struktur in Richtung des Zentrums unserer Galaxie gefunden. Die Signifikanz beträgt 2.52σ , was als konsistent mit zufälligen Fluktuationen betrachtet werden kann. Um eine unabhängige Überprüfung der Ergebnisse vorzunehmen wurde der IC40 Datensatz analysiert, der von der IceCube Kollaboration veröffentlicht wurde. Bei dieser Auswertung wurde tatsächlich in dem Bereich, an dem die Struktur aus den ANTARES Daten mit dem für Ice-Cube sichtbaren Bereich überlappt, ebenfalls eine Überfluktuation festgestellt. Die so identifizierte Struktur hat eine Signifikanz von 2.14σ .

Da trotz diverser weiterer Studien anhand der vorliegenden Daten keine eindeutige Erklärung der Ergebnisse möglich ist, wird eine weitere Analyse empfohlen, die auf die gefundenen Ergebnisse spezialisiert sein kann und daher deutlich eindeutigere Erkenntnisse liefern könnte.

Unabhängig davon ist das erhaltene Ergebnis gegenwärtig die signifikanteste, räumlich begrenzte Hypothese für einen erhöhten Fluss hochenergetischer astrophysikalischer Neutrinos.

Abstract

The origin of high energetic cosmic rays has been puzzling since their discovery. Many theories about the sources of these cosmic rays also predict a flux of high energetic cosmic neutrinos. Recently, the existence of such a high energetic neutrino flux has been confirmed, but the location and nature of its sources remains unknown. The ANTARES neutrino telescope was built in the Mediterranean Sea, 40 km off the French coast near Toulon in a depth of 2475 meters to help answer this and other questions. It consists of a three dimensional array of 885 photomultiplier tubes that detect the Cherenkov light emitted by secondary particles, which are produced in interactions between neutrinos and nuclei in the water.

The identification and reconstruction of the observed neutrino events constitute challenging tasks. Parts of this thesis deal with algorithmic approaches to improve these tasks using pattern recognition. The first application is the suppression of undesired background by a classification algorithm. The second approach is the selection of the best available direction reconstruction for each neutrino.

The main focus of this thesis lies on a new method to evaluate the spatial distribution of the observed neutrinos. While most approaches test one specific hypothesis for a specific source, derived from theory or other measurements, this search refrains from optimizing for individual source hypotheses and tries to detect the most pronounced density fluctuation in the spatial distribution, regardless of its specific position, size, shape or internal distribution as unbiased as possible. To achieve this, the statistical likelihood for the observed neutrino density is evaluated in multiple scales up to distances between events of 180°. To recognize a potential cosmic neutrino signal, regions with the most pronounced deviations are identified and compared to the expectations from a random background hypothesis. The strength of such a flexible, model-independent search is not the sensitivity for a specific source hypothesis, but instead to detect also unexpected hypotheses that can then be analyzed in more detail.

In the data recorded from 2007 to 2012 this search found a very large structure close to the direction of the center of our galaxy with a post-trial significance of 2.52σ . It can therefore be explained best by a statistical fluctuation. As a simple crosscheck this method has been applied to a publicly available data sample recorded independently by the neutrino telescope IceCube. This evaluation also resulted in an overfluctuation at the location where the most significant structure from ANTARES data overlaps with the field of view of IceCube. With the devised analysis method the found structure in the IC40 data has a significance of 2.14σ . While this is intriguing, ultimately, a dedicated follow-up analysis that is optimized for the derived hypothesis is necessary to find unambiguous evidence for its true nature.

Since, despite further studies, no unambiguous explanation could be found for the obtained results, a follow-up analysis is recommended, that can be adapted specifically to the results and therefore has a higher chance to provide unambiguous insights.

Nevertheless this result constitutes the most significant spatially resolved hypothesis for the sources of high energetic astrophysical neutrinos so far.

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Part I Introductions

This part of the thesis introduces various basic concepts useful for the understanding of the methods and results. It covers aspects of neutrinos in general, the ANTARES experiment, the underlying physics and of the algorithms used.

1 Motivation

The sky has always been fascinating for mankind, making us wonder about the mechanisms behind the universe. Until the 20th century, observing the sky was limited to the visible part of the electromagnetic spectrum. This began to change in 1931 when Karl Guthe Jansky noticed that a part of the background noise in transatlantic radio communications originated from the Milky Way [1]. Over the course of the following decades the sky was studied not only in radio wavelength, but also in all other parts of the electromagnetic spectrum like microwaves, infrared, ultraviolet, X-ray or γ -ray. An example for the different aspects revealed by different parts of the electromagnetic spectrum is shown in Figure 1. Another good example for a multiwavelength observation revealing different aspects of the same object can be seen in Figure 5. With every new analyzed energy range new phenomena were discovered, deepening our understanding of the processes that shape our universe.



Figure 1: Our galaxy as seen in different wavelength bands. Taken from [2].

Unfortunately for astronomy, electromagnetic waves are absorbed if dense matter is between us and the source. This prevents a glimpse into some of the most interesting regions, for example the center of our galaxy.

A completely different approach in astronomy is to rely on other particles instead of photons. One candidate are so called cosmic rays. These particles have first been described by Victor Hess in 1912 in [3]. They consist mainly out of protons and alpha particles and their flux extends to very high energies as

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seen in Figure 2. It shows the flux of cosmic rays versus energy, measured by a multitude of independent experiments. For the low energy range, which is not covered in this plot, direct measurements of cosmic rays can be performed, for example by experiments on balloons, satellites or the International Space Station. The shown high energy range is mainly covered by Cherenkov telescopes, for instance the Pierre Auger Observatory [4]. But cosmic rays are charged particles and therefore they're influenced during their propagation by galactic and extragalactic magnetic fields. Hence only the most energetic particles are deflected little enough that they approximately point back to their origin.



Figure 2: The energy spectrum of cosmic rays. From [5]. Multiple experiments have measured different energy ranges with different techniques. The features that can be observed in this energy distribution are the so called "knee" at around 10^8 GeV and the "ankle" around 10^{10} GeV. The distribution has different spectral indices between them. These features suggest that different mechanisms could be involved.

The relatively new field of neutrino astronomy is a promising candidate to overcome these drawbacks. Neutrinos are electrically neutral particles, so they are not deflected by magnetic fields. They interact only by gravity and the short ranged weak force, allowing them to travel large distances even through matter. Because neutrinos cannot be observed directly, their detection must rely on secondary particles, which are generated once a neutrino interacts with matter. These secondary particles can emit Cherenkov radiation, which occurs when charged particles travel faster than the speed of light in the medium they are currently traversing [6]. This radiation can then be measured for instance by highly sensitive photomultipliers.

The "Astronomy with a Neutrino Telescope and Abyss environmental RE-Search" project (ANTARES, described in [7]) is a Cherenkov radiation based neutrino telescope, located in the Mediterranean Sea, 40 km off the French coast. Other collaborations, for instance BAIKAL [8] and IceCube [9], are also doing research in the promising field of neutrino astronomy, with ANTARES being the largest operational telescope in the Northern Hemisphere.

For the success of every current, complex experiment sophisticated simulation and data analysis software is absolutely essential. This thesis illustrates improvements of several steps in the data evaluation chain of ANTARES. These improvements are not limited to this experiment as only minor details are application specific, but the ideas behind them are general and can be applied to a multitude of scenarios.

The first improvement is an enhanced classification to separate neutrino signals from undesired background. The second presented idea is to efficiently combine multiple independent reconstruction algorithms using a classification algorithm to obtain a more accurate reconstruction result. The main part of this thesis is a new strategy to detect structures of arbitrary position, size, shape and neutrino distribution in the observed neutrino signal which are incompatible with a random background assumption. This method does not require a theoretical model of the neutrino emission.

Chapters 2, 3 and 4 sketch an overview over neutrinos, neutrino detection with ANTARES and pattern recognition, covering properties of neutrinos, possible scenarios for neutrino emission, the architecture of the neutrino telescope itself, its data taking and the software used for data processing and simulations.

The new methods that have been developed for this thesis are explained in detail in chapters 5, 6 and 8.

The results of the created data processing chain and their explanations are presented in chapter 9, while the interpretation of these results can be found in chapter 10.

2 Neutrinos

The existence of neutrinos has first been postulated by Wolfgang Pauli in 1930 to explain the continuous energy distribution of electrons in beta decays, see [10]. The first proof of their existence was found in 1956 with the Cowan-Reines experiment, see [11], when electron-antineutrinos produced by a nuclear reactor were observed by inverse beta decay. Since then many other experiments have investigated the properties of neutrinos. They are electrically neutral and therefore only interact via gravity and the weak force. This is also the reason why they are so hard to detect, as they do not interact with magnetic fields, not by strong force interactions and, similar to to photons, hardly by gravity, see [12]. Neutrinos are fermions as they have a half-integer spin. There are three known types of neutrinos, the so called flavors. Together with the three leptons e, μ and τ , the three neutrinos (and the respective antiparticles of each particle) form the leptons, a group of light elementary particles. An overview over the standard model of particle physics with leptons in green is shown in Figure 3. Each neutrino flavor is named after the lepton the neutrino produces in charged current interactions. Maybe the most remarkable characteristic of neutrinos is that they oscillate between the different flavor states, alternating between electron, muon and tau flavor. Therefore neutrinos can be expected to have a mass greater than zero, since from theory it is known that oscillation between the neutrino flavors isn't possible for massless neutrinos, see for instance [13], [14], [15] and [16]. Nevertheless their exact masses are still unknown. The mass of the electron-neutrino ν_e for instance is known to be below 2 eV from measurements of the beta decay of tritium [17]. The standard model of cosmology together with the interpretations of different observations give upper limits for the sum of the masses of all neutrino flavors of below 1 eV [18] or even below 0.23 eV [19].

Since no difference between neutrinos and antineutrinos will be relevant for any part of this thesis, antineutrinos will not be addressed separately, but are included when referring to neutrinos from here on.

2.1 Sources

Neutrinos of different energy scales are produced by different objects and processes. The first observed electron-(anti)neutrinos from radioactive decays in nuclear reactors for instance had energies of a few MeV. A well established extraterrestrial source of neutrinos is the sun [21], which is also emitting neutrinos with energies up to few MeV [22]. Until now the only identification of a neutrino source outside of our solar system has been possible during the outburst of the supernova 1987A. It occurred on 23. February 1987 in the Large Magellanic Cloud, a nearby galaxy. Three neutrino experiments that have been active at that time, Kamiokande II [23], IMB [24] and Baksan [25] in total observed 25 neutrinos from the supernova, also with energies of few MeV. But there are neutrino sources that reach orders of magnitude higher energies.

As already mentioned in chapter 1, Victor Hess discovered cosmic rays in 1912. On further investigation it was discovered that this flux of charged par-



Figure 3: The standard model of particle physics. Taken from [20].

ticles extends to very high energies as seen in Figure 2. But even today the sources of the highest energetic cosmic rays are still unclear. There are models for a variety of galactic and extragalactic objects that predict the acceleration of hadrons to highest energies, making these objects potential candidates for sources of high energy cosmic rays. If these theories are true, the same hadronic processes must also generate very high energy neutrinos. Once an accelerated proton produces a hadronic shower, the resulting charged pions can produce neutrinos as described in equation 1 (Branching ratio >99.9%) followed by equation 2. Charged kaons which are also produced in hadronic showers can produce neutrinos either by decaying to charged pions (28%), resulting again in equation 1, or directly to muons (64%), resulting in equation 2.

$$\pi^- \to \mu^- + \bar{\nu}_\mu \tag{1}$$
$$\pi^+ \to \mu^+ + \nu_\mu$$

$$\mu^- \to e^- + \nu_\mu + \bar{\nu}_e \tag{2}$$
$$\mu^+ \to e^+ + \bar{\nu}_\mu + \nu_e$$

The existence of such a high energy neutrino flux has recently been confirmed by the IceCube neutrino telescope, published in [26] and [27], but it has

2.1 Sources

not been possible to identify a source of the observed neutrinos.

In our galaxy there are several candidates for sources of high energetic neutrinos. A promising candidate would for instance be supernova remnants (SNRs). The shock front generated by the immense release of energy during a super nova explosion propagates into the surrounding interstellar medium. By repeated scattering in front of and behind this shock front, particles from the interstellar medium can be accelerated to high energies. This acceleration process is known as Fermi-acceleration. It is explained in detail in [28]. Probably the most famous SNR, the crab nebula, is shown in Figure 4.



Figure 4: Supernova remnants like the crab nebula could be a source of cosmic neutrinos. Taken from [29].

Apart from SNRs, also pulsars could be possible sources of high energetic neutrinos. Pulsars are fast rotating neutron stars which have strong magnetic fields, allowing them to accelerate charged particles powerfully.

Another possibility for galactic neutrino sources could be the jets of so called micro-quasars, stellar black holes of a few solar masses accreting matter for instance from a companion star.

The Fermi Bubbles are large extended regions in our galaxy that reach 50° above and below the Galactic Center, with a width of up to 40° . They were first described in 2010 in [30]. As their origin is not completely clear, the detection of a neutrino flux originating from these bubbles would help clarify their formation. A possible scenario could for instance be a jet produced at the Galactic Center, which could also accelerate hadrons and thus produce neutrinos.

On the other hand models of these potential galactic sources show that these phenomena can't produce the most energetic observed cosmic rays, as explained for instance in [31]. Therefore even more powerful mechanisms must be involved. As there are no such mechanisms known within our galaxy and the gyroradius of these particles would already be larger than the extension of our galaxy, the highest energetic particles are believed to be of extragalactic origin.

One of the most promising candidates would for instance be active galactic nuclei (AGNs), see [32]. AGNs have a massive black hole in the center of the host galaxy. When matter is accreted and consumed by the black hole, jets are formed orthogonal to the plane of the accretion disk. These jets could be an excellent method to accelerate particles to highest energies. An image of such an AGN can be seen in Figure 5.



Figure 5: The AGN Centaurus A as overlay of multiple wavelength measurements. Taken from [33].

Another interesting possible source could be gamma-ray bursts (GRBs). They are short but very intense outbursts of highly energetic γ -radiation. As they are distributed isotropically over the sky, it was realized quickly that they must be of extragalactic origin, see for instance [34]. The typical timescale of GRBs is rather short, with two different timescales being observed. Short GRBs have a lifetime of below 2 seconds, whereas typical long GRBs last up to minutes. After both types a longer afterglow can be observed. Possible explanations for GRBs include for instance the collapse of a massive star to a neutron star or a black hole. More insight on GRBs can be found for instance in [35] or [36].

There are several other extragalactic objects which might contribute to the production of high energetic neutrinos. A more complete list is given for instance in [31]. But besides the already suspected candidates, of course there is also the possibility that the involved mechanisms are still unknown or not fully understood yet.

2.2 Propagation

Once a neutrino is generated at its source, it is hardly influenced during its propagation through space. As they are electrically neutral, they cannot be deflected for instance by galactic or extragalactic magnetic fields along their path. As explained in [12], they are not influenced by gravitational effects significantly more than photons due to their small mass and high velocity. This makes them ideal candidates for astronomy, since, unlike photons, they can traverse dense matter. Cosmic rays on the other hand are charged particles and therefore, except for extremely energetic ones, are deflected by magnetic fields along their way, obscuring their origin.

Since neutrinos oscillate between the three flavor states and the typical traveling distance is far longer than the distances required for oscillation to occur at the considered energies, the expected flavor ratio of cosmic neutrinos on earth is 1:1:1, regardless of the original flavor(s) that have been produced at the source, see for instance [37].

2.3 Detection

The biggest advantage of neutrinos for astronomy, their low probability to interact between the source and the Earth, becomes the most relevant disadvantage when trying to detect them with a neutrino telescope. A direct detection of neutrinos is beyond the capabilities of todays technology. Therefore the detection of neutrinos has to rely on the secondary particles created when a neutrino interacts via the weak force with a nucleus of the matter it traverses. The deviation between the direction of the neutrino and of the produced secondary particle is small at the energies relevant for this thesis as shown in [38], page 43. For example the median angular difference betweeen the direction of a muonneutrino with an energy of 1 TeV and the muon resulting from a CC interaction is below 1°.

There are two possibilities for interactions between neutrinos and matter. The interaction can occur as a neutral current (NC) interaction with a Z_0 boson as a mediator, shown in Figure 6, or as a charged current (CC) interaction, with a W^+ or W^- as mediator and the charged lepton corresponding to the flavor of the neutrino as resulting secondary particle, shown in Figure 7.



Figure 6: Feynman graph for the deep inelastic neutral current neutrino interactions of all flavors. From [39].



Figure 7: Feynman graphs for the deep inelastic charged current neutrino interactions. From [39].

If the interaction occurs in a dielectric medium and the produced charged secondary particles move faster than the speed of light in this medium the so called Cherenkov radiation, described for instance in [6] and [40], is emitted at a characteristic angle Θ_E .

$$\Theta_E = \arccos \frac{1}{n\beta} \tag{3}$$

Equation 3 shows the dependence of Θ_E on the refractive index of the medium n and the velocity v of the traversing particle with $\beta = v/c$. c denotes the speed of light in vacuum. For the considered energies and seawater this angle is about 42°. A schematic of the emission of Cherenkov light is shown in Figure 8. This light can be measured to detect the neutrino and to reconstruct its properties.



Figure 8: Schematic of the emission of Cherenkov radiation along the track of a particle. S' marks the starting point where the interaction took place, S is the current position of the particle. Due to the movement of the particle, which is faster than the speed of the light in the medium, a cone shaped light front is formed. Image taken from [39].

3 ANTARES

As explained in chapter 2.3, secondary particles produced by neutrino interactions emit Cherenkov radiation, which can be detected. Photomultiplier tubes (PMTs) can be used to measure these photons. Since the probability that a neutrino interacts close to a PMT is low, a large volume covered by a high number of PMTs is favorable. Of course this volume has to be filled with an optically transparent medium to allow the photons to propagate to the PMTs. Since a neutrino interaction only produces few photons compared to a visible light source, as few other light sources as possible should be interfering with the measurement, making the deep sea or deep in the antarctic ice favorable options for the location of a neutrino telescope. From the position, time and amplitude information measured by the PMTs, the direction of the secondary particle and therefore of the neutrino can be reconstructed and its energy can be estimated . All modern neutrino telescopes follow this general scheme. At the time this thesis is written, ANTARES ("Astronomy with a Neutrino Telescope and Abyss environmental RESearch") is the largest operating neutrino telescope in the Northern Hemisphere. It is located in the Mediterranean Sea¹, 40 km from the French coast near Toulon at a depth of 2475 meters to partly shield it against muons produced in the atmosphere. It uses the water of the deep sea as optical medium for neutrino detection.

The international ANTARES collaboration consists of 32 institutes from France, Italy, the Netherlands, Germany, Spain, Russia and Morocco.

3.1 Detector layout

The PMTs measuring the Cherenkov radiation in ANTARES have a diameter of 25.4 cm. They are enclosed in glass spheres together with some necessary electronics. These spheres are called optical modules (OMs). Three OMs and some additional electronics are grouped together in so called "storeys". The OMs of each storey have a spacing of 120° between them to optimally cover the whole surrounding. The structure of a storey can be seen in Figure 9. Each OM is oriented downwards by 45° because about 10^{6} times more muons generated by cosmic ray interactions in the atmosphere than from neutrino interactions reach the telescope from above, resulting in a large background. On the other hand, only neutrinos reach the telescope from below, as they are able to traverse the earth. Therefore ANTARES has been optimized for upgoing muons. A schematic visualizing the possible paths of muons that can reach ANTARES is shown in Figure 10.

Twenty-five storeys² are attached to each line with a distance of 14.5 m between the storeys. The layout of ANTARES contains 12 of these lines that are anchored at the sea bottom, each held upright by the buoyancy forces of a buoy attached at the top of the line and of the OMs along the line. The lines have a mean distance of about 70 meters between them. The footprint of these lines can be seen in Figure 11. All together this setup results in an extension of

 $^{^{1}42^{\}circ}48'N, 6^{\circ}10'E$

²One line has only 20 instead of 25 storeys

3.2 Event processing



Figure 9: An ANTARES storey with the three OMs. Taken from [41].

about 210 by 210 by 350 meters and an instrumented volume of about $0.01 km^3$. An artists impression of the overall structure can be seen in Figure 12.

Since the whole setup is exposed to sea currents, the lines are not always aligned exactly vertically. This effect is compensated by a calibration of the telescope geometry. The measurement of the current geometry is performed every two minutes using acoustic emitters and hydrophones located at every fifth storey. With the help of other techniques, these measurements allow position calibrations by triangulation with a precision of < 10 cm, see [44]. The obtainable time resolution, also achieved by sophisticated calibrations, is at a level of 1 ns.

Many more details about the components, structure, calibration and operation of ANTARES can be found for instance in [7].

3.2 Event processing

ANTARES follows the "all data to shore" concept, meaning that all measurements above a certain low threshold are sent to a computer farm at a shore station via optical cables. The measurements are then called a "hit". Every hit has a timestamp assigned by the electronics. Therefore the hits generated by the same physical event at similar times can be grouped together by the data processing chain to be recognized as one "event", sometimes also referenced as a "frame".



Figure 10: Sources of muons in ANTARES. Atmospheric muons from cosmic ray interactions in the atmosphere reach the detector from above, but only from interactions of muon-neutrinos can muons reach it from below. Image taken from [42].

To filter out events that contain only noise, caused for instance by bioluminescence, algorithms analyze the incoming data before they are stored. There are many ideas about how these so called triggering algorithms or "triggers" can search for potentially interesting events. In ANTARES they rely on a preselection of the measured hits, where only hits above a threshold or in coincidence with other hits on the same storey are analyzed further. The trigger algorithms then analyze these preselected, so called L1 hits. If any of the simultaneously active trigger algorithms identifies an event as potentially interesting, it is written to disk together with the hits contained in a time window of ± 2200 ns before and after the first and last triggered L1 hit. In this thesis two trigger algorithms are used for data selection.

• 3N trigger:

It searches for L1 hits that fulfill equation 4:

$$|\Delta t| < \frac{|\Delta x|}{c_{medium}} \tag{4}$$

with Δt being the time difference between two hits, Δx the distance between the position of these two hits and c_{medium} the speed of light in seawater.

If a cluster of at least five connected hits that fulfill this criterion is found, the hits are checked if they are compatible with a muon trajectory. If this is the case, the event is triggered by the 3N trigger.



Figure 11: Footprint of the lines in the ANTARES telescope. Taken from [42].

• T3 trigger:

At least two of the L1 hits are detected on neighboring storeys within 100 ns, or on next to neighboring storeys within 200 ns. If an event contains two of these coincidences, the T3 trigger keeps the event.

More information on the trigger algorithms can be found for instance in [45], [46] and [47].

The data taking in ANTARES is organized in so called "runs". A run typically lasts several hours and is intended to be a data taking period of similar conditions. The runs are identified using a runnumber, which is a six digit number. Until today for every run the first digit of the runnumber is zero. This leading zero will be omitted when listing runnumbers in this thesis.

Since one storey doesn't provide enough information to reconstruct an event decently, but low-energetic events often do not produce enough light to be observed by multiple storeys, a neutrino telescope has a lower limit for the energy of reconstructable events depending on the geometry, the data taking conditions and the applied reconstructions. For ANTARES this lower limit lies somewhere around 10 GeV, depending on the exact evaluation methods.



Figure 12: Overview over the ANTARES neutrino telescope. Based on a depiction from [43].

3.3 Event signatures

With the achievable resolution of ANTARES one cannot distinguish all possible types of neutrino interactions from each other. With three neutrino flavors, two possible interactions per flavor and one antiparticle per particle, in principle there are twelve different interactions. Since there is no way to measure the charge of the generated particles, the interactions of particles and antiparticles look alike. From the remaining six interactions all neural current (NC) interactions produce indistinguishable hadronic cascades, also called hadronic showers. This is indicated in the overview in Figure 13, with only four different signatures remaining. For energies relevant in ANTARES the signature of all particle cascades is almost pointlike. This can be seen in Figure 14, where the traveling distances for showerlike events are short, keeping in mind that the distance between two lines is about 70 meters in ANTARES. To resolve those structures a much denser detector is necessary, like for instance Super-Kamiokande, see [48], but these detectors do not yet cover such large volumes. With the dense instrumentation of future projects like ORCA, described in [49], it will be possible to resolve such small structures even for large volumes.

The charged current (CC) interactions of neutrinos produce the lepton corresponding to the flavor of the neutrino, accompanied by a hadronic shower. The electron created by ν_e CC interactions produces an electromagnetic cascade with a pointlike signature that is so similar to the ones generated by hadronic cascades that these cannot be distinguished with ANTARES for individual events. Therefore the interactions c) and d) in Figure 13 can be considered to have the same signature.



Figure 13: Possible event signatures for a neutrino telescope. Taken from [50].

In contrast to that, muons created by ν_{μ} CC interactions travel considerable distances depending on their energy as seen in Figure 14. With a typical energy loss, the track length of muons between 100 GeV and 1 TeV can already surpass the instrumented volume of ANTARES.

Since they emit light along their path, their signature is elongated and tracklike. Therefore muon-neutrinos can be distinguished from the showerlike event signatures. The signature of ν_{τ} is highly energy dependent. For energies below 100 TeV these events look pointlike since the τ has a lifetime of $(290.3 \pm 0.5) \cdot 10^{-15}$ s (see [17]) and therefore decays quickly, producing a second cascade within or next to the first hadronic one, making it similar to the ν_e signature. The expected traveling distance increases rapidly with energy at about 50 m/PeV, theoretically resulting in a tracklike event signature for highest energies. In between these two cases, a so called "double bang" signature is expected. Due to the size of ANTARES, it is unlikely that both cascades can be observed within the detector volume and can be distinguished from an ordinary track event with an additional energy loss due to a shower along the track. Furthermore, a nu_{τ} can also result in a track if the produced τ decays to a μ , which occurs in about 17.4% of all events, see [17].

This leaves us with two distinguishable signature types, the pointlike shower events and the elongated tracklike events as seen in Figure 15.

Due to its elongation the tracklike signature contains more information about the direction and therefore also allows a more precise direction reconstruction. The small size and the symmetrical shape of pointlike cascades render a direction reconstruction especially tricky. Although there has been substantial progress on this topic in recent years, see for instance [39], no competitive level is reached yet. Therefore this analysis uses tracklike signatures only.

3.4 Backgrounds

The desired signals are highly relativistic upgoing muons generated by cosmic muon-neutrinos. Besides a background level of about 3 kHz per PMT by elec-



Figure 14: Travelling distance for different events in water as a function of energy. Taken from [50].

tronics [42], an omnipresent background for this measurement in ANTARES are photons emitted by the radioactive decay of $^{40}_{19}$ K. This isotope is contained in natural sea water and, by the decays explained in equations 5 to 7, produces electrons with energies above the threshold for Cherenkov radiation.

$${}^{40}_{19}\text{K} \to {}^{40}_{20}\text{Ca} + e^- + \bar{\nu}_e \qquad (89\%) \tag{5}$$

$${}^{40}_{19}\text{K} + e^- \to {}^{40}_{18}\text{Ar} + \nu_e \qquad (11\%) \tag{6}$$

$${}^{40}_{19}\text{K} \to {}^{40}_{18}\text{Ar} + e^+ + \nu_e \quad (0.001\%) \tag{7}$$

This decay produces rates of about 37 kHz per PMT. As it can only be detected in the direct vicinity of OMs, this process generates uncorrelated background patterns and is therefore suppressed by the requirement of correlations on more than one storey.

Another omnipresent source of background is bioluminescence caused by multiple species of microorganisms as well as larger animals in the deep sea. An example for one of the larger culprits can be seen on the left in Figure 16. While the amount of light produced by the decay of $^{40}_{19}$ K is very steady, the



Figure 15: Actually distinguishable event signatures for a ANTARES. Modified from [50].

amount of bioluminescence is highly variable and influenced by different factors for instance the sea current, as shown in Figure 17. In some cases the light pattern caused by bioluminescence range over multiple storeys, but the pattern and the timing is clearly different from what is expected for tracklike event signatures. More information on the involved bioluminescence phenomena can be found in [51].



Figure 16: Examples for bioluminescence in macroscopic animals, here a comb jelly (left, from [52]) and an arctic comb jelly (right, from [53]).

Muons from interactions of cosmic rays in the atmosphere produce the largest amount of triggered events, as they are orders of magnitude more frequent than muons from neutrino events, see Figure 18. This problem may seem like it can be solved entirely by only considering upgoing muon tracks with a zenith angle³ $\Theta_Z > 90^\circ$ (or $\cos(\Theta_Z) < 0$ in Figure 18).

Unfortunately there is a small chance that the hit pattern produced by downgoing muons looks similar to an upgoing muon-neutrino event and there-

³Chapter 3.7 gives a brief introduction of the coordinate systems.



Figure 17: Environment measurements from the deep sea and background rates in ANTARES. There is a correlation observed between the optical rate in blue and the sea current in black. Taken from [51].

fore they happen to be misreconstructed by the reconstruction algorithms. Although the probability for this is tiny, the shear number of those events makes misreconstructed downgoing muons one of the main backgrounds for data analysis. Two of the approaches that are explained later, especially in chapter 5, deal with this problem. Depending on the accuracy of the filtering, the remaining sample contains mostly upgoing muon-neutrino events.

This selection is dominated by upgoing neutrinos from interactions of cosmic rays in the atmosphere on the other side of the Earth, which constitute an irreducible background for the search of cosmic neutrinos. As a first order approximation, these remaining background events are distributed isotropically in each direction. While deviations from this isotropy are known, overall these are too small to be relevant in this thesis. Furthermore, in the evaluation method described in chapter 8, the numbers are derived from recorded data and hence, to a certain extent, can take potential anisotropies into account as explained in chapter 9.7. An analytical model for the flux of neutrinos can be found for instance in [55] and [56].

Since the number of events expected from interactions in the atmosphere is considerably larger than the number expected from any cosmic source, an evaluation is required that recognizes the cosmic origin of neutrinos, for example based on features like their energy or spatial distribution. This task is the main focus of this thesis and will be addressed from chapter 8 onwards.

3.5 Software

The official software framework of ANTARES is called Seatray, described e.g. in [57] and [58]. It is derived from the software framework Icetray, which is



Figure 18: Angular distribution of the flux of muons above 1 TeV at the ANTARES location and depth. Atmospheric muons dominate the downwards direction almost up to the horizon. Taken from [54].

used by the IceCube collaboration. The framework is composed of several core components which provide the necessary basic functionality and multiple so called modules. These modules can be developed completely independent from each other by different users, each for an own analysis or other purpose. Since all modules are required to implement the interface defined by the Seatray core, they can be shared and combined at will. The core is written in C++, while users are free to write their modules either in C++ or in Python.

The feature extraction and classification used in this thesis is contained in a module called antares-rdfclassify. The functionality which is behind this module is also available as a stand-alone version called SGClassify. The multiscale source search is contained in a module called signal-first, which has been the name of this search during development. These modules can be found in the svn version control system of Seatray.

3.6 Simulations

To be able to evaluate the performance of data analysis algorithms in detail one would need a large sample of events for which all relevant information (Particle type, interaction position, energy, direction, for some evaluations even the produced secondary particles, ...) has to be known. Since it is virtually impossible to obtain this without any errors for recorded events, precise simulations have to be conducted, generating events where this information is available. This is achieved by stepwise Monte Carlo simulation (MC) of all involved processes. More detailed information on this topic than what is presented here can be found in [42]. 1. Event generation

A tool called GENHEN is used to generate neutrino events according to the expected flux. Documentation on the internals of GENHEN can be found in [59] and [60]. The outcome contains the trajectory of the primary particle (the neutrino) as well as long lived secondary particles (e.g. muons). The energy distribution of the events is expected to follow a power law with a spectral index of approximately -2.0 for cosmic neutrinos. As the simulation of these energy distributions would result in little available statistics for events with high energies, a harder energy spectrum is simulated and the events are assigned a numerical weight to be able to reweight each event according to any desired flux afterwards. The details of this weighting process are described for instance in [61] or [42].

The background of atmospheric muons with a spectral index of -3.7 is simulated using a tool called MUPAGE, see for instance [62]. The propagation of the particles to the telescope is simulated by a code called MUSIC [63].

2. Photon emission and propagation

The Cherenkov light emitted by all particles generated by the previous step is calculated by a program called KM3. As the tracking of each generated photon would be too intense computationally, a lookup table is used to model the generation, propagation and scattering of the photons between the track of the particle and the PMTs detecting them. This lookup table is based on fullscale simulations conducted with GEANT.

In addition to that, there is light produced by particle showers which occur along the tracks of the high energetic particles. To simulate these particle showers a tool called GEASIM is used. Internally it relies on GEANT for the simulation of the generated light, too.

3. The detector response

The outcome of the previous step are all photons that arrive at the detector. The next step is to simulate the behavior of ANTARES. The tool used for this task is called TriggerEfficiency. Information about the internals of this tool can be found in [64]. It simulates the response of the PMTs to the incoming photons and the effect of the currently active calibration on the electronics. The detected simulated hits are then processed the same way as measured hits as described in chapter 3.2.

To obtain simulations that better reflect the actual data taking and detector conditions, the simulation chain has been adapted to include the exact calibrations of the ANTARES neutrino telescope which were actually used during the simulated data taking periods. This also includes the duration of individual runs, conditions like e.g. the bioluminescence rate and the software setups. These individualized simulations are called run-by-run simulations (RbRs). There are two different versions of these simulations used in this thesis. The difference between RbR 2.2 and RbR 3.0 are the included versions of the

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used simulation tools. This thesis is written during the transition phase from version 2.2 to version 3.0. While in principle RbR 3.0 offers the best available simulation, RbR 2.2 are still used whenever there were not enough RbR 3.0 simulations available to guarantee a reliable evaluation.

3.7 Coordinate systems

Three different coordinate systems are especially useful for the understanding of the argumentations and results presented in this thesis.

First of all, there is the local coordinate system of ANTARES. It is defined by two angles, zenith Θ and azimuth Φ . The zenith angle ranges from 0 to 180 degrees. A zenith angle of 0° denotes a downgoing direction, directly from above ANTARES. A zenith of 90° is a horizontal direction and a zenith angle of 180° corresponds to a straight upgoing direction from below the telescope. The azimuth ranges from 0° up to 360° and specifies the rotation around the z-axis. An azimuth of 0° denotes the easting direction, 90° northing. Event reconstructions for example give their results in this coordinate system. Figure 19 shows the final event sample, obtained as explained in chapter 7, in local coordinates.



Figure 19: Distribution of the selection of neutrinos described in chapter 7 in local coordinates of ANTARES.

The second relevant coordinate system in this thesis is the equatorial coordinate system. In contrast to the local coordinate system, which revolves with the Earth, a fixed point in the sky, for instance a distant star, has fixed, unique coordinates in the equatorial coordinate system. The two angles that identify a point in this system are called declination δ (from -90° to +90°) and right ascension α (from 0° to +360°). Equatorial coordinates are constructed such that all points on a projection of the equator of the Earth on the sky have a declination of 0° . The direction straight above the north pole has a declination of $+90^{\circ}$, straight "below" the south pole is -90° . Analogous to the azimuth in local coordinates, the right ascension defines the rotation around the z-axis. Using equatorial coordinates to map the sky has the useful property that the visibility of ANTARES is approximately constant for all regions that have the same declination, making it the coordinate system of choice for most of the evaluations in this thesis. The distribution of the final event sample in equatorial coordinates is shown in Figure 20. Instead of the right ascension in degrees, this degree of freedom is often given as the so called hour angle, in hours, minutes and seconds, ranging from 0 to 24 hours. The exact coordinates of an object slightly vary over time due to the nutation and precession of the rotation of the Earth, and over larger time scales also due to the movement of the objects themselves. Considering the resolution of current neutrino telescopes, these tiny changes aren't relevant for neutrino astronomy (yet).



Figure 20: Distribution of the selection of neutrinos described in chapter 7 in equatorial coordinates.

The third important coordinate system is the galactic coordinate system. Similar to equatorial coordinates, a fixed object in the sky has unique fixed galactic coordinates, too. The two angles are called galactic latitude b (-90° to +90°) and longitude l (-180° to +180°). Instead of being oriented at the equatorial plane of the Earth like equatorial coordinates, galactic coordinates are based on the galactic plane, which is the plane the galactic disk revolves in. An object with a galactic latitude of 0° is located exactly in this plane, so many galactic objects have a latitude close to 0°. A positive latitude defines objects "above" the galactic plane ("above" corresponding to north on earth), negative latitudes are below the galactic plane. A galactic longitude of 0° corresponds to the direction of the galactic center, $\pm 180^{\circ}$ defines the rotation to the left and to the right. A schematic of the construction of galactic coordinates can be seen in Figure 21. Figure 22 shows the night sky in galactic coordinates. One can see nicely the benefit of this coordinate system for galactic astronomy, as the plane of our galaxy separates the upper and the lower part of the skymap. The distribution of the final sample of events in galactic coordinates is shown in Figure 23. A reference for the location of some nearby galaxies in both coordinate systems can be seen in Figure 24.



Figure 21: Concept of galactic coordinates. Modified from [65].



Figure 22: The location of the galactic plane in galactic coordinates. Modified from [66].



Figure 23: Distribution of the selection of neutrinos described in chapter 7 in galactic coordinates.


Figure 24: Locations of some nearby galaxies. a) in equatorial and b) in galactic coordinates.

4 Pattern recognition

Pattern recognition in a nutshell is the science of finding interesting structures in data. One of the simplest approaches to perform such a recognition is to apply a threshold to a variable, where events are considered signal above the threshold and background below. Of course in many applications, especially with complex data or more differentiated goals, one can improve considerably if a more sophisticated approach is used. This chapter introduces the basics of pattern recognition which are helpful for the understanding of the solutions presented in chapters 5 and 6.

4.1 Pattern recognition overview

Classical pattern recognition usually involves the individual steps as depicted in Figure 25:



Figure 25: The pattern recognition pipeline (here for images). These are the steps commonly involved in a pattern recognition solution. Taken from [67], original from [68]. Image credit: Thomas Wittenberg

1. Data acquisition

The first step in data analysis is the acquisition of the data. For a digital processing the data has to be sampled and quantized. Every day examples for this step are digital cameras or cell phones for images and speech. In the context of this work the PMTs and the electronics of the ANTARES neutrino telescope perform this task.

2. Preprocessing

Measurements usually contain noise or artifacts. Often these can be reduced at least partially, for instance by the application of filters to the data. Common examples include Gaussian and median filters, normalizations or edge detections.

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3. Segmentation

Segmentation means identifying the possibly interesting parts of the data for further analysis. While this may seem like a simple task, in many applications it turns out to be surprisingly difficult, yet also essential. The variety of methods ranges from simple thresholds in black and white images to complex algorithms, for instance the Color-Structure-Code [69]. For ANTARES multiple steps can be seen to fulfill parts of this duty, for instance hit selections or the triggers.

4. Feature extraction

Features are numbers describing the measured data in a way which is comparable between multiple recorded events. For images this could for instance be the size of an object found by segmentation. In many cases transformations can also be used to describe the data, for instance Fourier or Wavelet transformation. A typical example for a feature for ANTARES data would be the outcome of a reconstruction algorithm.

5. Feature selection

It is important to use meaningful features, since numbers which are uncorrelated to the current task don't benefit the classification and can even decrease the achieved performance. A tricky aspect is that sometimes features are hardly meaningful on their own, but allow to exploit valuable correlations in combination with other features. If the number of features permits an exhaustive search this would be a favorable option as it is guaranteed to find the global optimum. In cases where this is not feasible other approaches can be used, for instance feature transformations like the Principle Component Analysis (PCA) or the Linear Discriminant Analysis (LDA). In cases where a clear and broad global optimum exists without other pronounced (local) optima, even a simple greedy search approach can lead to the desired result. A more robust approach than greedy feature selection, that is also applicable to high dimensional feature spaces, is described in this thesis in chapter 5.4.

6. Training

Labeled data is data for which the desired outcome of the task is known. For all supervised learning algorithms it is required to train a model that maps the observed feature input to the desired output. The outputs are so called classes, integer numbers that represent one group of events that share a common property. A simple example that is used in this thesis is the separation in upgoing or downgoing events, considered signal and background. But also more complex assignments are possible like an estimation which is the best reconstruction algorithm for an event. Examples for such algorithms are artificial neural nets, decision trees or support vector machines.

Regression is an approach similar to classification, but the outcome is not an integer class number but a continuous value. This is especially useful for applications which try to estimate missing variables, but it is not used in this thesis. A different branch of methods would be unsupervised learning. These methods use similarities in the data to automatically identify classes. A widely known example is nearest neighbor clustering. Due to the requirement to identify certain, a-priori defined event signatures, only supervised learning strategies are considered in this thesis.

7. Classification

The trained model is applied to new data to estimate the outcome. A classification of labeled data which have not been used for the training can be used to evaluate the quality of the model and therefore to estimate the accuracy of the outcome on unlabeled data.

The preprocessing and feature computation are application specific for the different tasks in this thesis, but the classification algorithm is the same for all tasks and therefore its concept is briefly introduced here.

Much more information about various aspects of pattern recognition and the mentioned methods and concepts can be found for instance in [70], [71] and [72].

4.2 Random Decision Forests

There is a vast variety of classification algorithms available today, each with specific advantages and disadvantages. The selection of the classification algorithm for this thesis has been done based on a comparison of several algorithms as described already in [67]. The best performing method at that time has been the Random Decision Forest (RDF). Due to its favorable properties like a high classification accuracy, robust behavior, tolerance for correlated features and relatively fast execution times, the RDF algorithm has been used for all classification purposes in this thesis.

The concept of a RDF was first introduced in [73]. It is based on the decision tree model. A decision tree consists of so called "nodes". In the simplest and also most common case one node performs one decision on one feature. For instance it can test the value of feature number seven to be greater than 1.0. Depending on the outcome of this decision, a different linked node is traversed next. These linked follow-up nodes are called children nodes. Trees which are limited to two children per node are called binary trees. The first node in a tree is called root. Nodes without children are called leaves. These leaves can either simply contain one clearly assigned output (the class the event belongs to) or probabilities for different outputs. This structure allows to assign one output to one set of input variables. A simplistic example for a decision tree with two features F1 and F2 and two classes C1 and C2 is shown in Figure 26.

While this concept on its own is working well, many attempts to improve its performance have been made. The RDF tries to make use of the internal variation included in the data used for training the model. This is achieved by training an ensemble of multiple decision trees. But instead of using exactly the same data for each training, which would result in the same tree again and again, only a random subset of the training data and a random subset of the features is used for each tree. This produces decision trees that are able



Figure 26: A very simple example for a decision tree with two features F1 and F2 and two classes C1 and C2. The evaluation starts at the top. Each node represents one decision on one feature. The outcome of this decision determines which node to visit next. The rectangular final nodes, so called leaves, contain the result.

to generalize the contained information better than one single tree. The final decision which class is chosen for an event is obtained by a majority vote of the trees. The percentage how many trees agree on the class output can to a certain extent be used as a quality parameter. The implementation has been forked from the 2011 open source version of the alglib library, see [74]. If not stated otherwise, the parameters used for the training of the RDFs are set to 101 trees and for each individual tree a random subset of 60% of the events and of $2 \cdot \sqrt{n}$ features is used for the training, with *n* being the number of available features here. For the case of 804 features which is explained in chapter 5, this results in 57 features for each training. These values have first been optimized in a parameter study in [67] and since then some have been fine tuned further by observations on new datasets.

There are many algorithms for the training of a decision tree. Famous ones include for instance CART [75] or ID3 [76]. In general the optimization task is to determine which variable to use for the next split and where to set the threshold for this split. Since a globally optimal solution is very hard to compute (np-complex, see for instance [77]), most algorithms aim to generate "sufficiently optimal" trees by an iterative greedy selection. The training algorithm used in the implementation also follows this greedy selection scheme. To determine the next optimal split several metrics are commonly used, for instance the Gini impurity or the information gain. Further information on the properties and details of various training algorithms in general can be found in [75] and [76].

The RDF algorithm has some favorable properties for an application in this

thesis. For instance the highly popular class of boosted, tree-based algorithms also trains multiple trees, but increases the weights of events based on the classification result of previous trees. While this often achieves an even more accurate classification result in cases where the available labeled data represent the data in the application sufficiently well, it's performance is less stable if there are systematic deviations between the datasets. These can potentially occur when a classifier trained on simulations is applied to recorded ANTARES data. Compared to properly trained artificial neural networks (ANNs) the performance can be expected to be similar, but most tree based algorithms are less sensitive to imperfectly selected features. Strong correlations between the features or features without useful information for the current classification task can in the worst case prevent the training of an ANN from reaching a stable optimum (in a reasonable amount of time). Based on the achieved performance in many other applications⁴, the recent development of deep neural networks is considered a promising candidate to further enhance the solutions in this thesis involving classification.

4.3 Evaluation

When it comes to the evaluation of the performance of a classification one simple rule is to be followed at all times: Never evaluate on the same data that has been used for training. Otherwise the obtained numbers will be unrealistically good, but they do not at all represent the performance of the classification in a realistic application on unseen data. This can be avoided by a separation of the available data into a disjunct training and test set. The drawback of this method is, that events which are in the test set are never used for training and events in the training set are not evaluated. This can easily be solved by performing a second training and evaluation with interchanged datasets. These results already give a first impression of systematic effects influencing the model, but only with a "statistic" of two numbers. Additionally most algorithms benefit from more data used for training, so a model trained on the full dataset is likely to show a better performance than one trained only on half of the dataset. These considerations lead to the default evaluation method, cross-validation. It is the standard way to evaluate the performance of a classifier introduced in almost any book on classification, for example also in [71]. N-fold cross-validation splits the available dataset into n random, disjunct subsets. Then the first n-1 parts are used for training and the *n*th part is evaluated. The process is repeated n times, each time with one different subset used for evaluation and all remaining subsets combined as the training set. This guarantees, that every event has been considered in the evaluation, while a large fraction of the available dataset can be used for the training of the model each time. The ideal, extreme case would be leave-one-out cross-validation, where in each fold only one single event is evaluated. But this also means, that for a dataset with m entries also m repetitions of the training process have to be performed. While this constitutes the optimal case for smaller datasets, it

⁴See for instance comparisons at [78]

usually is neither computationally feasible nor required for large datasets. In this case a number of folds n < m is chosen such that the evaluation fits the requirements.

To assess the performance of the presented classifications some specific terms are useful. In this thesis the term efficiency of class X denotes the fraction of events that actually belong to class X and are correctly classified to be class X. If this efficiency is 1.0, all events of this class have been correctly identified. 0.0 means that no event of this class has been classified correctly. Purity of class X denotes the fraction how many of the events that are believed to be of class X actually are from this class. A purity of 1.0 means that no events from other classes were classified to be class X. 0.0 means that only events that actually are from other classes have been classified to be class X and none that actually belonged to this class.

In the case of only two classes, signal and background, purity and efficiency by default refer to the purity and efficiency of the signal class if not explicitly stated otherwise.

Part II Methods

This part presents the relevant aspects of the algorithms that have been developed for this thesis⁵.

All comparison plots are generated by discrete cuts. Therefore only marked data points in the Figures correspond to computed results. Lines connecting the points in some Figures are intended for visualization purpose only. Error bars show statistical errors if not stated otherwise.

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 $^{^5{\}rm Chapters}$ 6 and 8 are based on ANTARES internal notes ANTARES-PHYS-2015-001 and ANTARES-SOFT-2015-001, which are neither published nor publicly available.

5 Up / Down classification

5.1 Previous work

As discussed in chapter 3.4, the desired signal of upgoing muon-neutrinos is accompanied by a high number of background events of misreconstructed downgoing atmospheric muons. Common practice to get rid of these is to only include events with an upgoing reconstructed direction and to require a very good reconstruction accuracy. The idea explained here uses a prefiltering of events based on a classifier that distinguishes upgoing and downgoing signatures in the telescope. The intention of this classification is to allow a less strict cut on the required reconstruction quality, resulting in more neutrinos for analysis. The foundation for this classification has already been described in [67], but since then several optimizations have been introduced. The status was a classification using a RDF with two classes (up and downgoing) based on 137 features designed for this task. This achieved a mean suppression of downgoing muons of 90.8% and a mean efficiency for upgoing muon-neutrino events of 93%. The main problem with this result was that the number of misreconstructed atmospheric muons is orders of magnitude higher than the number of upgoing events. Therefore a suppression of 90.8% is not enough, especially since many of these events would also be suppressed by other cuts which are required for an analysis.

A modified version of this classification software has been used to investigate the possibility of a coarse energy reconstruction in [79]. It showed that the RDF classifier, together with the already designed and some task specific features, can successfully be applied to other tasks in ANTARES. This property is also observed for the classification approach described in chapter 6.

5.2 Optimizations

One of the first optimizations has been to extend the feature space. The features described in [67] all rely on one binning of the information belonging to an event in 100 time bins. The length of these bins varies as the duration of the events does, reducing the accuracy with which features based on this binning can be compared between different events. The feature computation has been extended to include the same computations as before in multiple different, fixed time binnings. The median time of all hits of one event is used as the center of these time windows, with a fixed time binning of ± 250 ns and ± 1000 ns for the feature computation. In addition to that, the number of hit selections used in the feature computation has been extended. In [67] the features were calculated twice, once on all hits and once on L1 hits only. A reminder of the meaning of these hit selections can be found in chapter 3.2. The updated version performs a third computation of all features on all triggered hits, so all hits that contributed to the activation of one of the triggering algorithms. New features from intermediate steps of the previous feature computations were also added for each of the nine feature computations (three different hit selections and three different timing windows). In total this results in 783 features (87 instead

of 68 features per computation). Besides these, externally computed features are also incorporated. These are the outputs of two standard ANTARES direction reconstruction algorithms, namely "Aafit", described in [38], and "BbFit", described in [80]. For each reconstruction algorithm the reconstructed direction and the various quality parameters are used as features. For the up/down classification used in this thesis the final feature vector for each event contains 804 features.

To address the fact that the number of signal and background events is vastly unbalanced (10^6 times more downgoing atmospheric muons), at one point during the optimization of this classification a cut was introduced that requires 80% of all decision trees of the RDF to identify an event as upgoing to actually accept it as upgoing signal. Events with an agreement below 80% were considered "probably still a downgoing muon" and therefore discarded. This variable is called "RDFSafety" in the data processing.

In total these changes improved the accuracy of the classification to reach a muon suppression of 97% while still preserving a muon-neutrino efficiency of 80%.

5.3 RDF cascade

As further optimization, this classification is now used as a two-step process to reach a higher muon suppression while preserving roughly the same amount of upgoing neutrino events. The second classification step can exploit the changed distribution of parameters due to the changed ratio of downgoing atmospheric muons to upgoing neutrinos and the specific properties of those downgoing muons that survived the first step. A further improvement was observed when the RDFs are not trained with the same number of events for both classes. In general the distribution of instances between the classes should mimic the distribution encountered in an application. But if the deviation between the classes, depending on the actual classification task, becomes too $large^{6}$, the training can become unstable and tends to ignore underrepresented classes. As the number of downgoing atmospheric muons in this task is tremendously larger than the number of upgoing neutrinos, the actual distribution was replaced. Asymmetric distributions for downgoing versus upgoing of up to 10:1 turned out to give stable results for the first classification step. To include a safety margin for later application, a ratio of 3:1 has been used for the training of the first RDF step. Therefore the first step identifies unclear events as downgoing

⁶ Too large in this case means that completely ignoring the class with fewer events leads to a smaller error. For example, in a two class classification where 99.9% of all events belong to class 0 and only 0.1% to class 1, the classification reaches 99.9% accuracy by labeling every event to be class 0. Therefore the training algorithm will optimize in that direction if the fraction of correctly classifiable events $(0.999 \cdot E_0 + 0.001 \cdot E_1)$ is lower. (E_x denotes the efficiency for class x, the probability that events from class x are recognized to be from that class.) So E_0 should be greater than $1 - \frac{0.001}{0.999} \cdot E_1$. If we optimistically assume that both efficiencies are equal (usually the efficiency of the smaller class is lower), then E_0 should already be greater than 99.9%. Since the efficiencies are more or less fixed for a task with given classes and features, there is a maximal stable deviation in the number of events between the classes. In reality the effect is a continuous trend, not a sudden change in behavior.

muons and can be seen as a course filtering to remove most of the muons. The training of the second step was performed with a ratio of 1:2, so with more neutrinos than muons. The effect is that the second step is very careful not to lose neutrino signal and is intended to only remove muons that can be identified easily due to the shifted distributions of the classes after the first step. For the given classification task the cascade with these distributions between the classes outperforms cascades with homogeneous or inverted distributions. This two-step cascade of RDFs allowed to drop the previously required cut of 80% tree agreement, but of course this can still be applied if needed.

The final achieved accuracy of this improved classification reaches a mean muon suppression of $99.85\% \pm 0.1\%$ (stat.) $\pm 0.02\%$ (syst.) and a mean neutrino efficiency of $81.7\% \pm 0.5\%$ (stat.) $\pm 0.8\%$ (syst.). The systematic errors here are estimated based on cross-validation results. They only account for the systematics of the classification, not those introduced by the detector, the simulations or the rest of the data analysis. It may not be intuitive to interpret the difference between the final RDF cascade result and the intermediate step. The 2.85\% gain in muon suppression (from 97% to 99.85%) may not seem like much, but it results in a remaining number of atmospheric muons of 0.15\% instead of 3%, which in the end means 20 times less atmospheric muons.

The result of this RDF cascade is shown energy resolved in Figure 27. To generate this plot, Run-by-Run Monte Carlo simulations version 2.2 have been used (See chapter 3.6 for more information on the simulations). It includes muon-neutrinos and atmospheric muons that were triggered by the triggering algorithms described in chapter 3.2. One can see that the efficiency with which muon-neutrinos are identified is relatively constant above an energy of 100 GeV. The suppression of atmospheric muons is very high for lower energies. The drop at higher energies is caused by the low statistics in the simulations for high-energetic events.

A comparison of cascade depths (numbers of RDF steps) and quality cuts showed that a second step is more efficient than requiring a higher RDFSafety value, but adding a third step is less efficient for this task.

5.4 Feature selection

An issue for the up/down classification in [67] was found to be the optimal selection of the features to use. The approach had been to use genetic algorithms for this optimization task. See for instance [81] or [82] for more information on their usage in optimization. Although in theory they could be flexible and directed enough to find an optimal solution in a reasonable amount of time, this approach turned out not to converge to any stable optimum. Since the feature space is high dimensional and the classification method is highly nonlinear, a greedy search that iteratively selects only the one feature that gives the highest gain in performance did not provide a stable solution either. In this context greedy searches starting with empty, full and partially filled feature subsets have been tested for addition and removal of features. An exhaustive search approach that simply tests all possible combinations of features is no option for computing time reasons here, as the feature space is large and the possible



Figure 27: Neutrino efficiency and muon suppression versus energy for the two step RDF cascade on RbR 2.2. The simulated events are weighted with an E^{-2} spectrum.

combinations are too numerous.

Therefore a different method of optimization one might call a "greedy ensemble" has been designed to solve this problem. The idea is to heuristically combine some of the selection criteria from genetic algorithms with the focused direction and short runtime of a greedy search algorithm.

The search starts with an empty solution. In the first iteration the empty solution is extended by one single feature that yields the best performance on its own. Since this decision must be based on a certain data sample, but might change for a different sample, the search for the optimal feature is performed N times, each time on a random subset of the training data. All different solutions that are generated by this process are kept. The number of new solutions per old solution cannot be greater than the number of tested subsamples N, and it is at least one in cases where all evaluations agree on the same feature. The result is an ensemble of M solutions, depending on how many features have been found to be the best possible addition for one of the previous subsets. M denotes the current number of solutions here. As an example the first iteration might result in M = 2 solutions with solution S1 containing feature F5 and solution S2 containing feature F8.

In the second iteration the same process is repeated for each of the M solutions that were found in the first iteration. For the given example the extension of S1 might lead to a new solution S3, which contains F5 and F8, a solution S4, that contains F5 and F6, and a solution S5, which contains F5 and F12. The same is done with the old solution S2, which could result e.g. in S6 with F8 and F5 and S7 with F8 and F1.

5.4 Feature selection

Although equal solutions like S3 and S6 can be treated as one solution, due to the exponential growth of the number of solutions this scheme quickly reaches a very large number of solutions to process in each iteration. Since testing tens of thousands of solutions in each step is not computationally feasible, a cutoff is applied at the end of each iteration. The set of all currently found solutions is reevaluated again on a larger fraction of the full data sample and only the K best performing solutions are kept for the next iteration. For solutions that have already been reevaluated multiple times, the mean of all evaluations is used, increasing the accuracy of the estimate over time.

This pruning of the list of found solutions keeps the runtime of each iteration almost constant also for large feature spaces. If K is set to 1, the scheme is mostly equivalent to a greedy search algorithm. In this case this approach inherits the main drawback of a greedy search, the tendency to get stuck at a local optimum. With a larger but fixed number of solutions per iteration this tendency gets reduced, but not eliminated. On the other hand, apart from effects due to hardware limitations like cache sizes, the runtime per iteration and therefore of the whole process scales linearly with the number of kept solutions.

This iterative scheme stops either when there are no more features to add, after a fixed number of iterations, or, and this constitutes the common case, until the obtained solutions did not improve the performance of the classification during the last L iterations. One could argue that no improvement in one new iteration should be sufficient to stop the optimization, but this would neglect the possibility that in some cases a gain in discrimination power can be achieved only by the correlation of multiple features.

As a further tweak the stopping of iterations is not done simultaneously for all solutions once the performance of the best solution doesn't improve anymore, but individually for each solution. The benefit of this strategy is that new solutions which are not yet as good as the current best one can still evolve further at least L iterations on their own, while solutions which are kept due to their good performance, but which do not improve any further can already be excluded from the iteration scheme to save time. As a result the optimization will continue to search for better solutions as long as any of the M solutions still improved during the last L iterations.

This algorithm shares traits with the genetic algorithms that have been used in [67], as it also uses generations of multiple solutions and tries to iteratively find improved new solutions derived from the old, but it also shares the fast and highly focused aspect of greedily searching for new features with a plain greedy search algorithm.

In the tested application scenario of up/down classification with a high dimensional feature space of 804 features, the described parameters by default are set to N = 3, K = 30 and L = 6. The number of currently found solutions M varies, but is smaller (in the beginning) or equal to K after each completed iteration.

The best solution for the full feature set on RbR 2.2 simulations has been obtained in a detailed study by Thomas Kittler [83]. The numbers for accuracy in this study do not consider the weights of the events in the simulation. The best solution⁷ found in this study for a plain up/down RDF classification reached a classification accuracy of $94.87\% \pm 0.42\%$ compared to $94.02\% \pm 0.42\%$ for the full feature vector. The relatively small gain can be explained by the observation that RDFs can handle strongly correlated features exceptionally well, a property they share with many other tree-based classification algorithms.

For the actually used two step RDF cascade as described in chapter 5.3, no solution with a statistically significant improvement of the performance could be obtained at all. The best solution⁸ in terms of a solution with few features but a good accuracy reached 96.67% $\pm 0.16\%$ in comparison to 96.74% $\pm 0.16\%$ for the full feature vector.

Therefore the RDF cascade in this thesis is used with the full feature vector as this classification scheme gives the highest accuracy that has been achieved by any means.

5.5 Possible improvements

An important improvement for an analysis would be to adapt the classification to the individual data selection. While the trained RDF cascade is a powerful tool to reduce the number of background events independent of the actual analysis goal and of other applied cuts, it could improve considerably if other cuts which are additionally applied to data in an analysis were also applied before the training. The reason for this is that many of the events which are filtered out by the classification would have also been rejected by other cuts, reducing the observed benefit of the classification. A training with accurately prefiltered datasets would also allow the classification to adapt to the desired application in the best possible way, giving the best final result.

Another idea how to potentially improve this classification would be to evaluate a relatively new branch of pattern recognition methods, often called deep neutral networks (DNNs). This contains for instance the so called deep belief networks, see for instance [84] or deep convolutional neutral networks, see [85] and various others. Comparisons⁹ between RDFs, several other classification techniques and DNNs for different tasks indicate the potential for a substantial gain in accuracy by DNNs.

 $^{^7}$ Using the features: 784,786,788,164,99,50,24,176,670,78,550,221,394,360,306,745,791,382, 686,295,748,271,101,495,111,192,320,124,424,265,91,755,737,622,290,425,52,611,609,679,510, 584,616,276,585,283,250,9,304,90,446,628,199,214

 $^{^{8}}$ Using the features: 784,788,786,252,581,438,494,27,12,36,50,791,342,686,78,513,360,285,75,694,138,236,390,304

⁹For instance at [78]

6 Selectfit

There are multiple direction reconstruction algorithms for tracklike muon events in ANTARES. The most common way to make use of these track reconstructions in a physics analysis is to evaluate the results of one favorable algorithm on all events and to ignore all others. For some events however the selected algorithm may not perform well or even fails, although these events would be reconstructed well with another algorithm. In these cases adding a different reconstruction avoids loosing these events and therefore benefits the final analysis. A first approach to combine algorithms could be to use thresholds in the form of:

- 1. If reconstruction A has a better quality parameter than a certain threshold, use reconstruction A.
- 2. If A has a poor quality parameter but B has a better quality parameter than a certain threshold, take reconstruction B.
- 3. Otherwise, do not use this event.

With some parameter scanning for the thresholds this approach can already give some benefit compared to a single algorithm. But on the other hand it is a time consuming process to manually tune the thresholds for every new application scenario. It also becomes increasingly complex if we want to use more than two reconstruction algorithms and sometimes it does not use the best reconstruction available for an event. An example for this non-optimal performance would be an event where reconstruction A has a quality parameter slightly above the threshold, but reconstruction B gives an outstandingly good reconstruction. The simple threshold approach would still use A here.

The idea of Selectfit is to have an automated process that cannot only combine many reconstruction algorithms, but can also choose the most accurate available reconstruction for each event as reliably as possible. Additionally, a new quality parameter for error estimation is introduced to allow the identification of well reconstructed events, regardless of which specific reconstruction was chosen.

6.1 Overview

Selectfit uses a classification to find the best reconstruction for a given event and also to estimate the error of the chosen reconstruction. The current version is designed to combine four reconstructions for tracklike events, namely Aafit [38], Bbfit [80], Bbfit with MEstimator [80] and Gridfit [86]. Another version exists which combines these four and the Dusj reconstruction, see [87] and [88], and the Q Strategy for shower events as explained in [89]. The first scenario is intended to enhance analyses using the muon-neutrino channel, whereas the second version explores the potential of this approach to combine tracklike and showerlike events in one single reconstruction. This would allow analyses to use events of both signature types at the same time. The results shown in this thesis are for the first, tracklike case only, because the handling of shower events in ANTARES is not yet at the same level of experience and precision as for track events.

The same strategy can also be used to combine energy estimation algorithms, but these ideas are pursued by other members of the ANTARES collaboration now and since energy information is not evaluated in this thesis, these attempts are also not covered here.

Detailed example scripts on how to use Selectfit for direction reconstruction can be found in Appendix A.1. Obviously, the logic behind Selectfit can also be used with any other classification algorithm, software package and similar application scenario.

6.2 Features for Selectfit

The current version of Selectfit uses the outputs of the algorithms it combines and the number of pulses as an energy correlated variable as input. For each reconstruction strategy it includes:

- The zenith value
- The azimuth value
- All available quality parameters

There are at least two quality parameters for each algorithm, but this is not necessary. One would be sufficient, but the more useful information an algorithm outputs about its result, the more accurate the selection and error estimation can be in the end. A detailed list of features can be found in appendix B.

6.3 Classifier for Selectfit

The classifier used here is again the versatile Random Decision Forest as already described in chapter 4.2. The classes in this task are not defined as upgoing and downgoing like they have been in chapter 5, but instead as the best direction reconstruction algorithm for each event for the first RDF and for a second RDF as the angular error of that reconstruction that has been selected in the first step. Selectfit consists out of these two steps.

6.4 Selecting the best reconstruction

The first and major step of Selectfit is to find the best direction reconstruction algorithm for each event. It is not necessary for all reconstructions to be available for an event. An algorithm may not output a result for instance if an internal optimization doesn't converge. If one or more of the algorithm results cannot be found, the angular difference of the missing reconstructions is internally considered to be 2π , hence this algorithm is not selected. The two predefined possibilities, four track reconstructions or four track and two shower reconstructions can be used in the current implementation as described in appendix A. The default outcome is a number ranging from 0 to the number of considered reconstructions:

- Class 0 : "No fit was available at all"
- Class 1 : "Aafit is the best available fit"
- Class 2 : "Bbfit is the best available fit"
- Class 3 : "Bbfit with MEstimator is the best available fit"
- Class 4 : "Gridfit is the best available fit"
- Class 5 : "Dusj is the best available fit"
- Class 6 : "Q Strategy is the best available fit"

The zenith and azimuth of Selectfit are set to the zenith and azimuth of the selected reconstruction.

Since the algorithm sometimes misclassifies an event, it can happen that the selected reconstruction is not actually the best that was available for this event. To achieve a better overall performance compared to the best single reconstruction algorithm, the error introduced by misclassification may not become larger than the benefit for events where a different reconstruction was chosen correctly. It has been observed that the misclassifications occur mainly for those events, where the best and the selected algorithm have a similar angular error. Therefore the majority of misclassified events introduces only small additional errors. Nevertheless, for the configuration with four track reconstructions, this prevented the addition of other, similarly designed reconstructions. If a reconstruction is too similar to an already considered algorithm or has a too poor performance, it will in total introduce a higher error due to more misclassification than what can be gained by the few events it reconstructs better.

An alternative to selecting one of the direction reconstruction results would be to merge all results, for instance with weights resembling how likely each reconstruction seems to be the best. This approach has been tested and typically produced a lot worse results than simply selecting the best. The reason for this observed behavior is that a good reconstruction with a low angular error will usually become worse by merging it with less accurate directions, even if the worse direction has a small weight. The case where two algorithms give a similar error, but lie on opposing sides of the true direction, is rare. Therefore merging of results is not contained anymore in the current version of Selectfit.

6.5 Error Estimation

If only one track reconstruction is used for an analysis, one would typically cut on the main quality parameters of this reconstruction in order to obtain a sample of accurately reconstructed events. But when combining multiple algorithms, cutting only on the quality parameter of the dominant algorithm will remove exactly those events, where the addition of other algorithms improved the result most. Combining the quality parameters by hand for each algorithm is the same task as combining the fits themselves. Therefore the chosen approach to solve this is again a classification. The main purpose of a cut on a quality variable of a direction reconstruction is to cut away those

reconstructions which have a large angular error. To mimic this behavior with a classification, the events are put into bins (classes) of similar angular errors during training. Some experiments on how these bins should be formed covered examples of linear inter-class distances (0°-1°, 1°-2°, ..., 179°-180°), exponential inter-class distances (0°-1°, 1°-2°, 2°-4°, 4°-8°, ...) and some manually hardcoded schemes. Since an ideal distribution for the training of this classification would have an equal number of events in all classes, also a dynamic computation of class boundaries with equal number of events was tested. The problem with this approach is that the class boundaries change for every training, especially if many classes are to be trained. Therefore this idea had to be discarded. For all remaining possibilities also the width of a class bin was varied as well as the start and end points (e.g. 0°-0.5°, 0.5°-1.5°, 1.5°-2.5°, ..., 44.5°-45.5°, greater than 45.5°). The experiments showed that an exponential distribution yields the best results. The exponential factor has some effect on the result, but its possible range is limited. It may not be too large because otherwise there are only few classes, which would only allow very coarse selection of events. It may also not be too small because then the number of classes grows quickly up to a point where not enough events for a stable training are contained in the less likely classes anymore. The final class distribution for the Selectfit error estimate was fixed to:

- Class $0 : 0.0^{\circ} 0.1^{\circ}$
- Class 1 : 0.1° 0.2°
- Class 2 : 0.2° 0.4°
- Class 3 : 0.4° 0.8°
- Class $4 : 0.8^{\circ} 1.6^{\circ}$
- Class $5 : 1.6^{\circ} 3.2^{\circ}$
- Class $6: 3.2^{\circ} 6.4^{\circ}$
- Class 7 : 6.4° 12.8°
- Class 8 : 12.8° 25.6°
- Class 9 : 25.6° 51.2°
- Class 10 : 51.2° 102.4°
- Class 11 : 102.4° 180.0°
- Class 12 : "No reconstruction available"

The quality parameter of Selectfit is set to the error estimate class determined by this classification. As for selecting the best fit, also for the error estimation some events are misclassified. Figure 28 shows the misclassification during error estimation. Ideally this would be a diagonal line. Figure 28a shows how well the estimation can work. In this test the same number of events has been used during training for each of the (in this study still 16 instead of 12) classes. The result of an optimization for the actual distributions is shown in Figure 28b. As can be seen in both cases, misclassifications occur dominantly between neighboring and next-to-neighboring classes. An increased probability for misclassifications between neighboring classes is not surprising, as this is for instance also caused by events with an actual angular error of 1.55° being identified as class 5 (starting from 1.6°) instead of the correct class 4 (up to 1.6°).



Figure 28: a) Normalized distribution for equal number of events per class during training, 16 classes. This scenario demonstrates how well the error estimation can be adapted in the the idealized case where each class has the same number of events. b) Distribution for a realistic, precut sample in absolute numbers, 12 classes, optimized to minimize errors for realistic distributions.

One idea to improve the result of the error estimation would be to use a regression, a similar method to estimate missing values, instead of a classification. The investigation of this possibility is pursued by another member of the ANTARES collaboration.

6.6 Accuracy of Selectfit

Combining the fit selection and the error estimation gives Selectfit. The results which are achieved for charged current neutrino events on the small available sample of RbR 3.0 simulations is shown in Figure 29. The x-axis shows the median angular error between the true simulated direction and the reconstructed direction. The different points in the figure are the results for different quality cuts, with stricter cuts being to left. The y-axis shows the cumulative efficiency, stating what fraction of neutrinos survive a certain cut level. The goal for a direction reconstruction is to reconstruct as many neutrinos as possible with an angular error as small as possible, so the best, only theoretically achievable point in this plot is to the top left. The red curve shows the performance for Aafit, the best and most often used single reconstruction algorithm. To obtain the different values of the curve a cut on λ , the main quality parameter of Aafit, was varied from -4.0 to -9.0 in steps of 0.1, which covers more than the commonly

used range. The blue curve is the performance of Selectfit on the same data set, with each point being a different cut on the estimated error class from 0 to 12. One can see that regardless of the actual cutting value, Selectfit does improve the performance of the direction reconstruction. The green curve is obtained by the "perfect Selectfit". For each event it chooses the best available reconstruction and it also uses the exact angular error of the selected reconstruction as error estimate. Therefore this is an upper limit on what could theoretically be achieved with the Selectfit approach for muon-neutrinos in ANTARES. For loose cut values and hence less accurate reconstruction results, the gain in the number of neutrinos by Selectfit is around 11%. For intermediate cuts, which is where e.g. a cut on λ greater -5.2 or -5.4 would be located, the gain is between 8% and 9%. This is between the 4.th and 5.th datapoint of Selectfit from the left in Figure 29. For the most accurate reconstructions both reconstruction methods deteriorate quickly, but Selectfit reaches about half the angular error of Aafit for the tiny, most precisely reconstructed neutrino samples. In total Selectfit shows a gain of at least 8% in the number of neutrinos for the whole range of possible quality cuts. Since the available RbR 3.0 sample has only limited statistics and has not yet been validated at the same level of detail, Figure 30 shows the same comparison for a larger sample of RbR 2.2 simulations. All plots showing results for RbR 2.2 do not include statistical error bars as they are indistinguishable from zero. This plot shows all events weighted according to an assumed signal flux of $E^{-2.0}$ and also includes Aafit with a cut on a second quality parameter β , which was set to the default optimized cut $\beta < 1^{\circ}$ as used in many analyses. One can also note that, even though the difference between the reconstructions is very similar, there is a difference observable for the absolute values of the median angular errors for both reconstructions. The better performance is achieved for RbR 2.2, which seems plausible since overall RbR 3.0 contains more realistic simulations, for instance scattered light from interaction vertices. Still it is not absolutely clear that the magnitude of this difference is fully covered by this effect, but since it can be observed similarly for both reconstructions, it is not a feature introduced by Selectfit and it will be investigated by future work on the simulations.

Figure 31 shows the individual causes of the improvement. It includes a curve in purple for always using Aafit as reconstruction but with a RDF classification as quality parameter and a curve in black which uses Selectfit to choose a reconstruction, but then again Aafit λ and β as quality cuts. This clearly shows that for high efficiencies and worse median angular errors the performance improvement is due to the selection of different fits. On the other hand for very accurate reconstructions, Selectfit and Aafit using an RDF error estimate have approximately the same performance. This shows that the gain for small angular errors is mainly due to the RDF error estimation.

A similar behavior can be seen in Figure 32, where the effect of changing the selection or the error estimation from Selectfit to the respective "perfect" step from the perfect fit is shown.

The contributions of the individual fits to the Selectfit results derived from recorded data without any cut on the error estimation are:



Figure 29: Comparison of Aafit with a varying cut on λ , Selectfit with varying error estimation cut, and the perfect selection of the best available fit with a varying cut on the exact angular error on RbR 3.0. Error bars indicate statistical errors.

- Aafit: $27.0 \pm 0.02 \%$
- Bbfit: $2.0 \pm 0.006 \%$
- Bbfit with ME stimator: 2.0 \pm 0.006 %
- Gridfit: 69.0 \pm 0.04 %

The errors for these numbers are statistical errors only.

The contributions of the individual fits to the Selectfit results in the final data sample for this thesis as described in chapter 7 are:

- Aafit: 48.2 \pm 0.6 %
- Bbfit: $0.5 \pm 0.07 \%$
- Bbfit with ME stimator: 4.7 \pm 0.2 %
- Gridfit: 46.5 \pm 0.6 %

As a further improvement for this application scenario one could also drop Bbfit without MEstimator, as it would reduce the probability for misclassifications, resulting in a more accurate final result.

Another idea to improve the reconstruction beyond what is shown in this thesis would be to adapt the training of Selectfit to the analysis it is used for.



Figure 30: Comparison of Aafit with λ and β cuts, Aafit with λ cut only, Selectfit and a theoretic, perfect Selectfit on RbR 2.2

If any other cuts that can be applied before Selectfit in the data selection of the analysis are also applied in exactly the same way to the training data, the resulting Selectfit models can be expected to achieve better results for an analysis using these cuts than without this adaption.

Summarizing the observations for the developed and tested version, Selectfit for the direction reconstruction of tracklike events outperforms the best available single track reconstruction algorithm for any compared accuracy and neutrino efficiency.



Figure 31: Improvement from the single steps of Selectfit in blue compared to Aafit with λ and β cuts in red. Using Selectfit to combine all algorithms but using λ and β to identify the most accurate reconstructions gives the black dotted line. There is only a gain for less strict cuts. The purple dotted line shows Aafit with a RDF to identify accurate reconstructions. A gain is present for very accurate cut levels only.



Figure 32: Theoretical improvement from perfectly error-free steps instead of trained Selectfit RDFs. The actual Selectfit performance is shown in blue. Replacing only the RDF selection of the best algorithm with an error-free selection gives the red dotted line. Replacing only the estimation of the angular error with an error-free estimation yields the purple dotted curve. Replacing both steps by the error free versions gives the green curve.

7 Data selection

7.1 Run selection

The considered time period is from the beginning of the data taking with ANTARES on 29.01.2007 until on 31.12.2012. To obtain a reliable and well understood data set, several criteria were required for data taking runs to be included in this analysis:

- The mean optical rate must be below 300 kHz. The purity and efficiency of some reconstruction algorithms drops considerably for even higher background rates.
- The run setup must be a physics setup. This is to exclude e.g. data with artificially emitted light, which was intended for calibration.
- The data quality category must be 1 or better. This category is derived as a standard quality assessment in ANTARES for all runs. More information on the exact procedure can be found in [90].
- The run may not be identified to contain so called "sparks". These are seldom malfunctions where light is produced by an electric discharge.
- The data file must have been processed correctly by the standard ANTARES data production and have been accessible on the mass data storage.

Most of these requirements are de facto standard for ANTARES data analysis. A list of all included runs can be found in appendix H. In total this corresponds to an effective lifetime of the detector of 1332.8 days.

7.2 Event selection

The data in the selected runs does not only contain upgoing muon-neutrino events, but mostly atmospheric muons. To identify the desired signal some filtering of the events is performed:

- Application of the up/down classification as described in chapter 5 to get rid of a large fraction of downgoing atmospheric muon events. Only events which are classified as upgoing (Class 1) are processed further.
- Selectfit as described in chapter 6 is applied to each event. Only events with a reconstructed zenith angle of more than 90° (upgoing) are kept.
- The estimated angular error from Selectfit must be below 1.6° (only error classes 0, 1, 2, 3 and 4 as defined in chapter 6.5 are kept).

Even after these cuts the resulting muon-neutrino sample is expected to contain a contamination due to the vast number of atmospheric muons. This chapter presents a comparison of the actual measurements with the event numbers and composition expected from simulations. But as explained in chapter 8, absolutely no information derived from simulations is used in this analysis.

7.3 Comparison of simulations with data

To check whether the expectations derived from simulations are valid for recorded data, a small sample of data runs has been analyzed together with the corresponding RbR simulations. The subset of data files used for this comparison is called "burn sample". It is excluded from the actual evaluation to avoid a potential bias. Some of the simulated contributions had to be scaled up to compensate for missing or empty files. This introduces an uncertainty that is not contained in the presented error bars.

Figure 33 shows the comparison of the small available sample of RbR 3.0 simulations with the corresponding data files. On the x-axis is the error estimation class as defined in chapter 6.5. More accurate direction reconstructions are to the left. On the y-axis is the number of events. The simulated events in



Figure 33: Data Monte-Carlo comparison for RbR 3.0.

these plots have been weighted according to the expected atmospheric flux. The plot is cumulative as the values shown e.g. at error class 2 contain all events of classes 0, 1 and 2. Therefore these cumulative curves show the composition an actual sample would show for different cuts on the estimated error class.

Ideally the red curve, representing the event numbers from data, and the black curve, containing the sum of all simulated contributions, should match. Obviously this is not true for very poor reconstruction qualities as seen for large values of the error class in Figure 33.

This is not surprising since class 12 for instance means that there was no reconstruction result available at all, while classes 10 and 11 denote events with reconstruction errors between 51.2 and 180.0 degrees. Events that have such

tremendous reconstruction errors are likely triggered due to pure noise without any particle interaction, which is not included in the simulations.

Since no spatially resolved analysis could work with errors of up to 180 degrees, Figure 34 shows a zoom to the more interesting region of, depending on the goal of an analysis, potentially acceptable reconstruction errors. What can be seen here is that the agreement between data and Monte Carlo simulations is rather good for ANTARES standards. What is also visible is that for the used quality cut value of class 4 or better, this comparison suggests that there is hardly any remaining contamination of atmospheric muons left.



Figure 34: Data Monte-Carlo comparison for RbR 3.0. Zoom to realistic cut values.

But as the statistical uncertainty for the RbR 3.0 sample is large, another crosscheck with RbR 2.2 has been conducted. Figure 35 shows the same comparison as Figure 33 did, this time for a different sample of RbR 2.2 runs. The observed behavior is similar to the previous comparison, but the gap for very poor angular errors is even larger. Again following the reasoning that arbitrarily large angular errors can't be used for a spatially resolved analysis, the zoom to realistic values in Figure 36 also shows a good agreement between MC and data and mostly similar distributions as before on RbR 3.0. But it's important to note that in this comparison more atmospheric muons survive harder cuts, resulting in an expected contamination on the order of 29.7% for the remaining sample after a quality cut for error class 4. The applied Selectfit has been trained using RbR 3.0 and can be expected to give more accurate results on the more recent simulations. In the comparison using RbR 2.2, the remaining muons may as well be muons that have not been simulated correctly in this



Figure 35: Data Monte-Carlo comparison for RbR 2.2.

older version. Nevertheless, since a conservative estimate is preferred here and because the comparison on RbR 2.2 has higher statistics, this is also considered the contamination rate for the obtained final sample. Based on the numbers for a cut on error class 4 in Figure 36, the unblinding of the full data sample has been expected to yield 13078 \pm 114 events.



Figure 36: Data Monte-Carlo comparison for RbR 2.2. Zoom to realistic cut values.

8 Multiscale source search

The data recorded with the ANTARES neutrino telescope allows to address diverse physics questions. Many sophisticated analyses have been performed, each optimized specifically for one of the different possibly detectable physical phenomena, see for instance [91], [92], [93], [94] or [95].

Most searches for spatially confined sources of cosmic neutrinos rely on a model assumption derived from theory describing one specific potential source. These searches evaluate the distribution of reconstructed neutrinos to detect or reject this specific hypothesis. This usually means searching for an excess beyond the expectation by atmospheric neutrinos at a known position in the sky with a predefined size and shape.

The idea behind the model-independent multiscale source search is to approach this task the other way around. Instead of optimizing highly for one specific prediction, this approach is intended to reveal the most significant excess of neutrinos within the recorded data in any region in the sky. A dedicated analysis can then be optimized for the hypothesis derived from the findings of this search.

The main advantage of this approach is that, due to its unbiasedness and flexibility, it could potentially also reveal neutrino sources which would not be tested in a realistic time scale by other searches. Apart from that there is a highly reduced risk that this search suffers from deviations between assumptions, simulations and the reality.

Of course there are disadvantages which come with an evaluation like this. The main drawback is the high number of trials when evaluating for sources of arbitrary sizes, shapes and distributions on all positions in the sky. This inevitably lowers the sensitivity for any single specific hypothesis. But as already explained, the main objective of this search is to identify a promising hypothesis which can then be tested by a dedicated follow up analysis. Another negative aspect of such a strategy is that, compared to a search for one specific hypothesis, the interpretation of the result is less straightforward. Multiple plausible interpretations can be found, which may have to be tested in more detail. Finally, instead of identifying a source, an unspecific approach like this may also just reveal a yet uncompensated systematic effect in the measurement. While in terms of knowledge gain on cosmic neutrinos this could be seen as a disappointing result, it would definitely be useful for other upcoming analyses in ANTARES and maybe also similar experiments like KM3NeT [96].

Taking everything into consideration, the model-independent multiscale source search offers a good chance to identify the most dominant sourcelike structure in the data, while, on its own, it's unlikely to confirm such a structure with a high significance.

8.1 Input

Once the events are selected as described in chapter 7, their direction information is converted from local coordinates to equatorial coordinates (see chapter 3.7). This is done with the astro package from Seatray (see chapter 3.5), using only the reconstructed zenith, azimuth and the time the event was recorded. Everything that is actually required as input for the multiscale source search are the declination and the right ascension of each event. Evaluating energy and time too is possible in general, but not included in this search.

8.2 The search space: spherical grids

To evaluate the distribution of neutrino positions a discrete grid is used. This grid approximates a sphere by 165016 gridpoints which have a distance of \approx 0.5° to their next neighbors in declination as well as right ascension. The distribution of the gridpoints is 720 points at a declination of 0° , with a reduced number of gridpoints in other declinations according to the cosine of the angle between the declination and the horizon. This results in rows of gridpoints with less points per row towards the poles. This setup could be replaced by a more optimal scheme, which distributes the gridpoints more evenly, but the current implementation allows to exploit the regularity of the indices of gridpoints to speed up some lengthy computations considerably. The spacing is chosen to be 0.5° because a spacing of 0.25° drastically increases the runtime of the evaluation such that pseudo-experiments cannot be done in a reasonable amount of time any more. Furthermore this spacing corresponds nicely to the expected median angular resolution of about 0.44° achieved with Selectfit and the chosen quality cut. On the other hand using a wider spacing, for instance 1°, unnecessarily reduces the accuracy of the method. The radius of the spherical grid is set to 1.0 to simplify the computations. Figure 37 shows a spherical grid with gridpoints in blue and random events in white¹⁰. The tiles at each gridpoint have a radius of about 0.5° , hence they overlap and form a closed sphere. The upper part without neutrinos is north (declination of $+90^{\circ}$) in equatorial coordinates. The distribution of the events follows the visibility for and acceptance of ANTARES as described in chapter 8.3.3. There will be one of these spheres used separately for the evaluation of each of the 180 distance scales discussed in chapter 8.3.2. According to the sampling theorem as discussed in [100] or [71], using a grid with a spacing of 0.5° theoretically allows to reconstruct information of the scale of 1° or larger without error. This limit is shifted for this grid due to the discontinuities of the gridpoint spacing to around 1.5° . This doesn't mean that smaller scales can't be resolved in this search, but that they can't be guaranteed to be resolved without discretization errors due to the binning.

The distance of two points on the sphere is computed as the great-circle distance, the shortest distance between two points on the surface. To minimize numerical errors the more common equation 8 was substituted by equation 9 for computing the great-circle distance. Here ϕ denotes the declination, λ the right ascension and $\Delta \sigma$ the angle between point 1 and point 2. Since the radius

¹⁰For the visualization of the search spheres and the events on them, the vtk file format [97] is used. The visualization of the vtk files is done with Paraview 4.1 [98]. The skymaps in later chapters are plotted with Gnuplot, mainly Version 4.6 patchlevel 5, see [99].



Figure 37: A spherical grid with gridpoints in blue and random events in white. The gridpoints are rendered with a radius of 0.5° , hence they overlap and form a closed sphere. Since the sphere is a three dimensional model, only the front part facing the observer is visible here.

of the spheres is set to 1, $\Delta \sigma$ is also directly the distance measure.

$$\Delta \sigma = \arccos(\sin \phi_1 \sin \phi_2 + \cos \phi_1 \cos \phi_2 \Delta \lambda) \tag{8}$$

$$\Delta \sigma = 2 \arcsin(\sqrt{\sin^2(0.5\Delta\phi) + \cos\phi_1 \cos\phi_2 \sin^2(0.5\Delta\lambda)}) \tag{9}$$

8.3 From neutrinos to search spheres

To demonstrate the steps of the multiscale search we look at a distribution of random events with two artificial point sources at a declination of -70° . The setup is shown in Figure 38. To illustrate the steps more clearly the sources are unrealistically strong, one with 18 neutrinos within an area of 0.5° by 0.5° , the other one with 12 neutrinos.



Figure 38: a) A spherical grid with 12000 random events and two point-like sources. View from below to the south pole (declination of -90°). b) The same setup displayed without the random events.

8.3.1 Counting neutrinos

The search starts by counting the number of neutrinos located in a ring around each gridpoint with a radius corresponding to the current search scale. The first distance scale is from 0.0° to 0.5° .

for each gridpoint in grid:
for each neutrino:
 if 0.0 < distance < 0.5:
 counter = counter + 1</pre>

For an illustration of this counting scheme see Figure 39. It shows the counting for a search scale of 1.0° to 1.5° . The result of this evaluation is one number for each gridpoint of the search sphere. The results for three scales from the example with two point sources from Figure 38 can be seen in Figure 40.

8.3.2 180 spheres

The process described in chapters 8.3.1 to 8.3.6 is performed for 180 different search scales (from 0° to 90°) in steps of 0.5°. Each iteration *i* evaluates only the neutrinos in a ring (not the whole circle area) between $i \cdot 0.5^\circ$ and $(i+1) \cdot 0.5^\circ$ around each gridpoint, with index *i* starting at 0. The result of each iteration is computed independently (except for the smoothing between search distances in chapter 8.3.4) and stored in an independent spherical grid.

8.3.3 Expectation from pseudo-experiments

The next step will be to compute the Poisson probability for the observed number of neutrinos at each gridpoint. To compute this probability one needs the expected number of neutrinos. The value how many neutrinos are expected



Figure 39: Scheme of the neutrino counting. Crosses mark the gridpoints with a distance of 0.5° between them. Green and red dots are neutrinos. The red cross is the gridpoint that is being evaluated. The current search scale is between the black circles. In this example it is between 1.0° (inner circle) and 1.5° (outer circle). Neutrinos which are found in the current search scale at the current search point are shown in red. In this example the outcome of the evaluation of this scale at the red gridpoint is 13.

around each gridpoint is defined by the visibility of a point in equatorial coordinates for ANTARES, folded with the acceptance of ANTARES for the local coordinates that contribute to these equatorial coordinates, folded with the efficiency of the software reconstruction chain for these local coordinates. Even a complex analytical model is only an approximation for the effects of all these contributions. As an alternative the expectation can be approximated directly from the measured data, which is done in this search. This automatically includes the properties of the acceptance and efficiency of the whole reconstruction chain by design. Additionally, such a strategy doesn't rely on simulations to be "precise enough", nor does it need any manual tuning if for example parts of the reconstruction chain are exchanged.

To obtain the expectation the set of all neutrinos as described in chapter 8.1 is used again. If we were to have abundant statistics and only background neutrinos without any neutrinos from sources, the measurement itself would already be an approximation of the expectation. Since there are only few neutrinos and some of them could originate from spatially confined sources, the data is scrambled first using random times within the data taking period when



Figure 40: a) The spherical search grid with the number of events counted in a ring between 0.0° and 0.5° around each gridpoint. b) Number of events between 3.0° and 3.5° . c) Number of events between 10.0° and 10.5° .

computing the equatorial from the local coordinates. Potential source structures in the right ascension are removed by that, placing all events randomly in a declination band. Then the numbers of neutrinos are counted as described in section 8.3.1.

The problem remains that there isn't enough statistics to cover the whole sky with the 165016 gridpoints with the few thousand events. To improve this one can simply compute the mean of the counted values in one declination band. This is justified because in theory the expected number of events in equatorial coordinates should be the same at each point within one declination band, since the rotation of the Earth is supposed to obliterate any possible inhomogeneities. This assumption is by far exact enough at this point to estimate the expectation, but, as described in chapter 9.7, it is not 100% exact. The result is shown as the red curve in Figure 41. Still, these mean values per declination band show a high fluctuation due to the low statistics, especially near the poles where only few gridpoints belong to one band. To reduce the fluctuations of these values, several filters are applied to the vector. These filters are well known for instance from the field of digital image processing as explained in [100], [101] and [71]. The concept of filtering to reduce fluctuations from noise has been used before its application to digital data. The effects of filters on signals of all kinds have been studied extensively in the more general field of signal processing, explained for example in [102] and [103]. The values of the first two bins are ignored by the filtering algorithm in this case due to the large unphysical fluctuations around the poles.

The first applied filter is a so called median filter, a non-linear filter that for each value of the vector vec_i at the index *i* computes the median of the values $vec_{i-1}, vec_i, vec_{i+1}$ and stores it as the new value for vec_i . This is a common approach for instance in image processing to remove artifacts. The result of this processing can be seen in the green curve in Figure 41.

The resulting expectation still contains an unreasonable amount of statistical fluctuation with a high frequency. Therefore the next step is to use a linear low-pass filter. For linear filters the stencil notation will be used. This notation is the common way to write down discrete linear filters, see for instance [100].



Figure 41: Approximation of the acceptance of ANTARES and the reconstruction chain for the search distance between 0.0° and 0.5° .

For the vector of means and one dimensional data in general, a generic stencil looks like this:

weight [w c e]

Applying this stencil to a value of the vector at index i, with i being the index of the declination band here, means computing equation 10:

$$value_{(i)} = weight \cdot (w \cdot value_{(i-1)} + c \cdot value_{(i)} + e \cdot value_{(i+1)})$$
(10)

Using such a separable linear filter multiple times equals the one-time usage of a larger filter, for example:

twice 1/4 [1 2 1] equals 1/16 [1 4 6 4 1]

A brief explanation for this example can be found in appendix C. The benefit is that the larger filter doesn't have to be implemented manually.

In the two dimensional case, for instance when the points of the search sphere are considered to be pixels of an image, a linear filter looks like this:

The value is computed for the gridpoint at the center of the filter, again denoted with c here. Applying the stencil to a gridpoint at index i, j, with i being the
index of the declination band and j the index of the right ascension within the declination band, means computing equation 11:

$$\begin{aligned} value_{(i,j)} &= weight \cdot (nw \cdot value_{(i+1,j-1)} + n \cdot value_{(i+1,j)} + ne \cdot value_{(i+1,j+1)} \\ &+ w \cdot value_{(i,j-1)} + c \cdot value_{(i,j)} + e \cdot value_{(i,j+1)} \\ &+ sw \cdot value_{(i-1,j-1)} + s \cdot value_{(i-1,j)} + se \cdot value_{(i-1,j+1)}) (11) \end{aligned}$$

The filter which is used here to reduce the statistical fluctuations of the vector of expected mean values is one of the simplest one-dimensional low-pass filters:

1/3 [1 1 1]

This corresponds to a simple mean of the three values. All of these filters need to treat the borders in a special way, simply because there is no value left of index 0. Due to the spherical nature of this task the best value at the index -1 (which would be one before the first index 0) is the value of index 1, because this is also the value corresponding to the position of the other side of the pole. The stencil for index 0 is then

1/3 [0 1 2]

For the last index it is the other way around. Simply using this filter would efficiently reduce the statistical fluctuations, but it treats all values as if they were produced with the same statistics. To compensate for this effect an additional tweak is used here. For indices where one of the values is 0.0 the low-pass filter is not applied. This preserves the location of the boundary where the visibility and efficiency drop to zero and therefore prevents distortions which would occur at the tail otherwise. This filter is applied 60 times to obtain a continuous approximation of the expected mean. As explained earlier this corresponds to a one-time application of a larger lowpass filter. The filter is also applied once without the treatment for zero values to include the uncertainty for the measurement for values close to zero. The result of this procedure is shown in Figure 41 as the blue curve. To further reduce fluctuations which could be introduced by the scrambling of the data, the whole process of obtaining an expectation value for each gridpoint is conducted 30 times and for each gridpoint the mean of the 30 computed values is used.

Since this expectation is different for every search distance that is evaluated, this process is performed separately for each of the 180 search distances. The results for 10° to 10.5° are shown in Figure 42. One can clearly see that, due to the increased statistics for larger search rings, the statistical fluctuations are a lot smaller and the approximation is more accurate.

The obtained values of these approximated expectations are then set for every gridpoint according to its declination, resulting in expectations for the spherical grids as shown in Figure 43 for 0, 10 and 50 degrees. The magnitude of the expected values differs as already seen in Figures 41 and 42, therefore the color scale (corresponding to the y-value in the two-dimensional plots) is readjusted here. Red are high values corresponding to an expectation of more event, blue are low values indicating the expectation of fewer events from this direction.



Figure 42: Approximation of the acceptance of ANTARES and the reconstruction chain for a search distance between 10.0° and 10.5° .



Figure 43: The distribution of the expected number of neutrinos. a) 0.0° to 0.5° around each gridpoint. b) 10.0° to 10.5° . c) 50.0° to 50.5° . The color scale is rescaled.

8.3.4 Poisson probabilities

The algorithm has counted the number of neutrinos n around each gridpoint as described in chapter 8.3.1 and has an estimate for the expected mean number λ from chapter 8.3.3. With this information the Poisson probability P for n could be computed using equation 12, where e is Eulers number.

$$P(n) = \frac{\lambda^n e^{-\lambda}}{n!} \tag{12}$$

The problem with this computation is that it is numerically unstable for large values of n or λ . This instability not only leads to slightly wrong probabilities, but even to the failure of the computation. A possible solution to this is to take care of the exponent and mantissa separately as in equation 13. Another idea is to use the datatype long double¹¹ and the corresponding library functions with a higher precision for the computations. But both of these approaches only shift the range of the instability to higher values, they don't get rid of it entirely. Combining the two optimizations shifts the value



Figure 44: Comparison of the implementations and approximations of equation 12 for $P(n = \lambda)$. A probability of -0.1 denotes a failed computation.

where the computation of $P(n = \lambda)$ breaks down from 27 for float and 144 for double precision to 1547. Although this is enough in most cases of this search,

 $^{^{11}}$ Using g++ (SUSE Linux) 4.3.4 on the Woodcrest cluster in Erlangen long double gives 128 bit precision.

an approximation using the so called "Stirling formula" as denoted in equation 14 is introduced for larger values.

$$P(n) = \frac{\frac{e^{-\lambda}}{\log_{10}(e^{-\lambda})} \frac{\lambda^n}{\log_{10}(\lambda^n)}}{\frac{n!}{\log_{10}(n!)}} \cdot 10^{\log_{10}(e^{-\lambda}) + \log_{10}(\lambda^n) + \log_{10}(n!)}$$
(13)

$$P(n) = \frac{e^{n(1+\log_{10}(\frac{\lambda}{n}))-\lambda}}{\sqrt{2\pi(n+\frac{1}{6})}}$$
(14)

Since the deviation of this approximation from the true values vanishes already around $P(n = \lambda = 20)$ it is used in all cases where either n or λ is above 120. The behavior of the different implementations is shown in Figure 44. The final result using long double precision for computation gives exact results in terms of double precision and has been confirmed numerically stable until $P(n = \lambda = 10^7)$, which is definitely enough for much larger searches.

The algorithm then computes the probability for the observed or a greater number of neutrinos using equation 15 at each gridpoint.

$$P(x \ge n) = 1 - \sum_{i=0}^{n-1} P(i)$$
(15)

For historical and technical reasons the Poisson probabilities are then rescaled as in equation 16.

$$R = \log_{10}\left(\frac{1}{P(x \ge n)}\right) \tag{16}$$

To reduce statistical (only few events) and systematical (event reconstructions have errors) fluctuations, the R values are low-pass filtered on the whole sphere. The assumption is that "real" sources are more likely to extend to neighboring gridpoints, whereas for random fluctuations the probability to have another random fluctuation at a neighboring gridpoint is low. The stencil used for this filtering is:

Two dimensional filters have been explained in chapter 8.3.3. This filter focuses strongly on the central value for each gridpoint, adding only 40% of neighboring information. This is motivated heuristically, because it preserved the signal in various test cases. Finally a filter is applied to smooth between the different search distances. The motivation for this is that a source which is prominent in a search distance of for example 1.0° will likely be observed when searching for 1.5° too, while random fluctuations are less likely to extend to neighboring search distances. For each gridpoint the neighboring values are from the searches with a 0.5° smaller and 0.5° larger search ring. The stencil used is:

 $1/6 \ [1 \ 4 \ 1]$

The effect of these computations on the search spheres is shown in Figure 45. Red means high values, blue means low values. The color scale is readjusted between the different search scales.



Figure 45: a) The spherical search grid from Figure 40a.
b) The grid after the computations described in chapter 8.3.4.
c) and d) analog for the search radius between 3.0° and 3.5°.
e) and f) analog for the search radius between 10.0° and 10.5°.

8.3.5 Segmentation

At this point of the analysis spheres have been computed where high R values at a gridpoint indicate more neutrinos in a certain distance around the gridpoint. Therefore potential source regions containing more neutrinos should be linked to high values on the spheres. We can assume that low values are not linked to detectable sources. In pattern recognition separating background from potentially relevant information is called segmentation. A lot of information on segmentation can be found in [100]. The chosen solution for this task in this analysis is probably the simplest possible solution, applying a threshold to all R values. Similar to the segmentation of a black and white image, a histogram of the values of all gridpoints is analyzed to obtain the threshold. The distribution as shown in red in Figure 46 has a long tail of non-zero values, which is where potentially signal-related information is contained. If only the maximal values



Figure 46: The histogram of the R values on the grid, the distribution of only high R values and the smoothed distribution of high values.

are considered, the search still shows a good potential to find the most dominant source, but a lot of information on the shape of the source and on other less dominant sources is lost. To extract the tail of the distribution robustly, the first step is to cut away the lower 50%. The result is shown in green in Figure 46. Only very low values close to 0 are discarded by this. The next step is a smoothing of the histogram 50 times with the stencil

1/3 [1 1 1]

The result of this filtering is shown as the blue curve in Figure 46.

In the next step a line is calculated from the point where the smoothed distribution of the upper 50% has its maximum value maxVal to the first point on the right side where the y-value is below $0.5 \cdot maxVal$. The x-value where this line intersects y=0 is used as a threshold θ . This is illustrated in Figure 47. Only values in the tail right of this threshold are kept, lower values are set to



Figure 47: A fit from the maximum trough the half-maximum point determines the threshold.

zero. The advantage of computing the threshold like this compared to a fixed threshold or a fixed percentage is that the threshold adapts to the distribution. The effect of the segmentations on the search spheres is shown in Figure 48.

An additional option, which will be used later, is to scale the distance between the previous minimum value x_{min_old} and θ , using a factor of α . The new threshold is then given by equation 17.

$$\theta_{final} = (\alpha \cdot x_{min_old} + (1 - \alpha) \cdot \theta) \tag{17}$$

For now alpha is set to 0 and therefore $\theta_{final} = \theta$.



Figure 48: a) The spherical search grid from Figure 45b. b) The grid after the segmentation described in chapter 8.3.5. c) and d) analog for the search radius between 3.0° and 3.5°. e) and f) analog for the search radius between 10.0° and 10.5°.

8.3.6 Remapping

At this point the search has obtained spheres where only potentially signal related gridpoints have values different from 0.0. The next step is to reconstruct the location of the neutrinos that caused the detected overfluctuations. For the search sphere with a search distance between 0.0° and 0.5° the situation doesn't change, since the information up to 0.5° is already located at the exact gridpoint it originated from when counting the neutrinos. For all other spheres with search distances $d > 0.5^{\circ}$, the information stored at a gridpoint originated from counting neutrinos that are d degrees away from the gridpoint. To evaluate where the overfluctuation actually occurred, this information must be mapped back to the origin. This is done using a copy of the current spherical grid with all gridpoints initially set to 0.0. Then the following steps are performed¹²:

```
for each gridpoint p_{(i,j)} in the original grid:
find the set p_d of all gridpoints in distance d around p_{(i,j)}
for each gridpoint p of p_d in the new grid:
value_p = value_p + \frac{value_{p_{(i,j)}}}{size(p_d)}
```

Afterwards the new grid contains the corresponding fractions of the overfluctuations mapped back to their origin and this grid is used from there on. The effect of these computations on the search spheres is shown in Figure 49. The remapping distributes the information of an overfluctuation at one gridpoint back to all gridpoints where the neutrinos causing the overfluctuation could have been. The information where the neutrino distribution had a higher density is therefore automatically encoded in the pattern how the remapped circles overlap, see for example Figure 49d. As already stated in the beginning, only marginal differences are observed for the first grid as seen in Figure 49b. The color scale for the remapped plots on the right is not readjusted between the search scales. One can see that the point sources are detected most dominantly by the contributions from smaller scales.

¹²The naive implementation of this step, as in the explained scheme, has an extremely long runtime. Therefore the actual implementation differs slightly from the explained scheme to exploit a much more cache-coherent access pattern. In other words: The optimized remapping performs the same operations in a more efficient ordering.



Figure 49: a) The spherical search grid from Figure 48b. b) The grid after the remapping described in chapter 8.3.6. Nothing changes up to 0.5°. c) and d) analog for the search radius between 3.0° and 3.5°. e) and f) analog for the search radius between 10.0° and 10.5°.

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8.4 180 to one search sphere

The current status of the search are 180 spheres, each representing the search for overfluctuations in the number of neutrinos at a different scale. Each of these searches is left with only the most relevant results for its distance.

A reasonable next step would be to search for structures in these 180 spheres, taking into account the neighbors within a search sphere as well as the neighboring search spheres. This has been implemented with two different clustering approaches, but none could be optimized well enough to robustly and reliably find the test sources. There were multiple issues with these approaches, for instance when to optimally stop a cluster from growing in which direction (within one sphere must be treated differently than between spheres), how to map the three-dimensional clusters to a two dimensional skymap in the end and several more. For all of these problems there were solutions found, but the search didn't reach a satisfactory performance.

The better and feasible solution turned out to be to fuse the 180 search spheres into one. It is not a priori clear what value to assign to each gridpoint in the resulting sphere when we have a vector of 180 values. Several methods how to do this have been tested, for instance assigning the

- maximum value
- median value
- mean value
- sum of all values (L1 norm)
- L2 norm of all values (see equation 18)
- L3 norm of all values
- number of connected non-zero values
- and several others.

The Ln norm for a vector v is defined as:

$$L_n = \sqrt[n]{\sum_{i=0}^N v_i^n} \tag{18}$$

with N being the number of entries in the vector. Although taking the maximum value might seem like a reasonable idea, this is not robust as many gridpoints were observed to have a random overfluctuation somewhere in the 180 values. What worked best in terms of robustness and source detection for the scenarios tested during development was the simple sum and the L2 norm of all values. The reason is that significant sources tend to be found on multiple scales, resulting in many relatively high non-zero values in the sum. Random fluctuations on the other hand can often be observed on a single scale only. The ordinary sum was chosen for highest robustness and a good ability to identify source locations. During this optimization it was discovered that not including the first evaluation bin (from 0.0° to 0.5°) in the sum further enhances the stability of the overall search. Therefore it is not included.

It should be noted that the potential of this step could be exploited even better, either by evaluating the full 180 spheres or by a more sophisticated way of combining the 180 to one. But the devised algorithm is the best achieved solution. The result of the summation can be seen in Figure 50.



Figure 50: The sum of the 180 spheres

8.5 From one search sphere to clusters

Now there is only one search sphere left containing the combined information on neutrino density from all search scales. A source could now either be a very high value on that sphere or a larger region with "higher-than-usual" values or a combination of both. To identify these regions where "higher-than-usual" values are present, a segmentation can be performed exactly as described in chapter 8.3.5. This time α is varied, resulting in different numbers of remaining gridpoints. $\alpha = 1$ means not cutting away anything at all, a lower α means a harder cut. $\alpha = 0$ is the same threshold as used for the individual spheres, but also harder cuts with negative values are possible. One can see the effect of different thresholds for segmentation in Figure 51. Higher positive values introduce additional artifacts but also allow extended source structures to be found. Hard negative cuts only preserve the highest peaks, which works well for the presented unrealistically strong point sources. Another illustrative example for the effect of different segmentation thresholds can be found in appendix D.

Because all values of the whole sphere are taken into account for the segmentation, this is a global approach. In contrast to that also local approaches that focus on the change of the value next to a cluster (for example via edge detection and heuristics) or the mean and variance of the values from within a cluster and the surrounding neighborhood of the cluster have been tested. The problem with these local methods was that faint random clusters in neutrinopoor regions had an unreasonably high chance of surviving the segmentation, whereas relatively obvious extended sources without clear contours were overlooked often. The segmentation could certainly benefit from exploiting the local structures in a reliable way instead of only using the global distribution, but the global approach is the best found solution that performs well and is guaranteed to show robust behavior.

This step can introduce a bias to the search. Cutting softly results in keeping many gridpoints for the rest of the search, which prefers extended sources. Cutting harshly on the other hand biases towards smaller sources and removes faint but large sources completely from the remaining sample. Therefore multiple segmentations are used and evaluated independently. This introduces an additional trial factor that has to be accounted for in chapter 8.6.2, but it also allows the search to continue as unbiased as possible. Nevertheless, due to the trial factor, not too many different segmentations can be kept for this analysis.

The next step applied to the segmented grid is a non-linear median filter. This is a common approach in image processing to remove artifacts as explained in [100]. Similar to the linear two dimensional filters already introduced in chapter 8.3.3, the median filter also operates on the current gridpoint and its direct neighbors. But instead of writing a weighted linear combination of the considered values to the central gridpoint, it writes the median of the nine values. The effect of this filtering is shown in Figure 52.

The gridpoints of the resulting segmented spherical grid are then checked for connectedness. A connected group of gridpoints is called a segment in the general context of segmentation and in this work it is also called a cluster. All gridpoints are checked in the order from highest to lowest value and each time the considered gridpoint is directly connected to an already found cluster, it is added to this cluster. Connected here means that one of its eight neighbors is already part of the cluster. If it is not connected to any cluster, a new cluster is started. Once all gridpoints are assigned to clusters, all found clusters which have directly neighboring points are fused together.

After this is done all remaining gridpoints are contained in clusters. They have the same extensions as the segments in Figure 52.



Figure 51: The effect of different α values on the segmentation result.



Figure 52: The effect of the median filter on the segmented grids.

8.6 From clusters to significances

At this stage the information is condensed to clusters. Each cluster is composed of a vector of the gridpoints that are part of it. The next step is to distinguish the potentially significant sources from random accumulations. To do this one needs to know the probability how likely a cluster could have been generated by random events. This probability could in theory be easily determined by pseudo-experiments using scrambled data. The problem with this approach is, that requiring *exactly* the same cluster at the same position with the same gridpoints and the same values for each gridpoint is certainly not going to happen in a bearable amount of time. And to compute a probability we would need many occurrences of exactly this cluster. So this is not an option.

8.6.1 Relevance of a cluster

We can however describe the clusters using various evaluation metrics. For instance one could compute the probability for a cluster of the same size or larger, with size measured by the number of gridpoints. With a fair amount of pseudo-experiments there will likely be a sufficient amount of smaller and larger clusters to estimate the probability for the size of an observed cluster. To make sure that this probability is not confused with the significance of a cluster, we call the value computed in such a way "relevance". It can be interpreted as an estimation of the pre-trial significance, but it uses approximations and implicitly already contains some trial factors for the search in multiple scales and locations. The purpose of the relevance metrics is to provide a measure with which the clusters can be compared with all other clusters from all segmentations.

Only considering size to measure the relevance neglects smaller or even point-like sources. A good metric to find these could be the maximal value of any gridpoint of the cluster. But also many other metrics can be defined to identify outstanding clusters, each with a certain, sometimes not obvious bias. An incomplete list of the tested metrics can be found in appendix E.

Apart from basic, directly computed relevance metrics one can also use meta-metrics that rely on other relevance metrics. This can be done for instance by direct linear combination of basic metrics. The idea behind these combinations is that a dominant source could stand out in multiple certain metrics. Taking this idea one step further leads to even more general metametrics, but at some point these computations become so variable that sources without a clear common trait can become hard to distinguish from random clusters.

If all relevance metrics were to be used for the evaluation of the found clusters, the potential to achieve a high significance would drop considerably, because for the metric that might actually identify a source with a high significance there is a large number of other metrics and each metric has a chance to overfluctuate for a random accumulation. But on the other hand if we chose too few metrics we might introduce a large bias and maybe miss a source in the data. Since the intention of this search had been not to optimize for a specific source model, the selection had to be done heuristically, loosely optimizing for all sources that have been included in the pseudo-experiments during the development.

While a combination of two metrics, metrics 0 and 1 in appendix E, was found to be performing very well for all kinds of sources as shown in chapter 8.7, only metric 0, the size of the cluster in gridpoints, has been used for the data analysis of ANTARES data to reduce the resulting trial factor. By this choice the search becomes biased towards larger extended sources¹³, but this has been a deliberate choice, justified by the fact that ANTARES has already conducted sophisticated searches for small and pointlike sources, see for instance [91]. It is relevant to note that the choice of suited metrics is influenced by the choice of segmentation thresholds from chapter 8.3.5 and vice versa. Together with this explicit bias towards larger clusters and the resulting choice for this metric, the segmentation thresholds α have been fixed to -0.11 and +0.25. A list of the tested setups that have been used to derive these choices is contained in Appendix G.

For this selection of metric and segmentations several thousand pseudoexperiments with scrambled data are conducted. Each pseudo-experiment computes the whole process as described in chapter 8 up to this point. Since one pseudo-experiment typically contains many clusters, this results in tens to hundreds of thousands of comparison values for each metric. But only the maximum value for any cluster in any metric is considered in each pseudo-experiment, so that in the final analysis a cluster will be compared to the distribution of the achieved maxima instead of the distribution of all values. This proved to be more reliable than comparing against the whole distribution, because there are different systematics for the highest relevance value of a cluster in each pseudo-experiment than for the second or third highest, since their possible relevance values can be influenced by the highest scoring cluster in this pseudoexperiment. The comparison with the distribution of the maximal values leads to a correct estimation for the most relevant cluster in each pseudo-experiment, but to an underestimation for clusters with lower relevance values in the same pseudo-experiment. This solution is used because the aim of this search is the most significant structure in the sky. The found clusters can now be compared in each metric with a distribution of values from pseudo-experiments.

In this context a few definitions are useful. The p-value denotes the probability how likely something occurs. At this stage for example a p-value of 0.2 for a cluster is interpreted that in 20% of the pseudo-experiments a cluster with the same or a higher relevance value has been found in the same metric. Significances are given in σ , which is derived from the standard deviation of a Gaussian distribution. Therefore $\approx 68.27\%$ of all observed values are expected within a 1.0 σ interval around the mean value for a Gaussian distribution, about 95.45% within a 2.0 σ interval and in general a fraction of $erf(\frac{N}{\sqrt{2}})$ within N σ . erf is the (Gauss) error function here. In the two-sided interpretation it is irrelevant whether a value is higher or lower than the mean value. Since this

¹³The search would still be able to detect unexpectedly strong small sources, but with a considerably reduced sensitivity. The evaluation of multiple scales results in an extended cluster area around the actual small location (see Figures 49d and 52b) for strong sources.

search is aiming for overfluctuations only, the significances are interpreted in a one-sided way, meaning that low values are ignored and only high relevance values result in high significances. Hence 1.0σ instead means that in 68.27% of the pseudo-experiments no relevance value as high as the observed one has been found. For example 2.52σ corresponds to a p-value of 0.0117, approximately one in 85, 2.14σ to 0.0324 or about one in 31.

Directly comparing a cluster to the exact obtained distribution of relevance values to compute the p-value only allows certain, discrete p-values to occur. This means that two clusters with different relevance values may be mapped to the same p-value if no value is between them in the distribution used for comparison, making their relevances indistinguishable. The possible minimal p-value would also be limited by the number of pseudo-experiments.

Therefore the tail of the distribution of relevance values is substituted by a fit, following an exponential decay in the form of $a \cdot e^{bx}$, which is the tail of a Poisson distribution. The result can be seen in Figure 53. The fit is computed based on the part of the tail of the distribution where the green fit is also shown in this plot, but it is only used for high values where the actual statistic is not sufficient anymore, indicated by the threshold value in black. Up to this value



Figure 53: An example for the distribution of relevance values and the fit to this distribution. The fit is computed based on the whole tail of the distribution, but it is only used for values greater than the threshold marked in black.

the actual distribution is used to evaluate the significance. The threshold in black, from where on the fit instead of the plain histogram is used, is calculated as $0.8 \cdot beginning_of_the_tail+0.2 \cdot value_where_third_to_last_zero_gap_occured$. If there are less than three gaps in the histogram with a zero value, the beginning of the first present gap is used instead. This heuristic is used to make sure that a range of the distribution with stable statistics is used to fit the tail, regardless

of binning effects and poor statistics.

After the comparison of the relevance values with these distributions each cluster has one p-value for every metric, expressing how likely it could have been produced by chance according to this metric. The overall p-value for a cluster is the minimum of the p-values from all metrics. From that p-value a renormalized relevance (similar to a significance) is computed using the inverse error function¹⁴.

8.6.2 Significance of a cluster

At this stage we have clusters on the search sphere, each cluster with a value called renormalized relevance which states how likely it could have been produced by a random neutrino distribution. The relevance would equal the significance if we had used only one relevance metric and only one segmentation. But there is a trial factor for the additional possibilities that the observed renormalized relevance value can be produced in any segmentation (and by any metric if more than one is used). The corresponding trial factor could simply be applied now, but since it is a conservative upper bound and the different segmentations are highly correlated, we can give a more accurate estimate by conducting pseudo-experiments. A large number of pseudo-experiments with scrambled data would have to be conducted again. Each experiment would have to compute the whole process described in chapter 8 up to this point again. But almost the whole process has already been computed in chapter 8.6.1 to compare the relevance values to the distributions from pseudo-experiments. The only additional step is the computation of the renormalized relevance values for each cluster. So the results from the previous pseudo-experiments are reused, the few missing computations are added and we have a large distribution of renormalized relevance values from pseudo-experiments.

Just as for the relevance values in chapter 8.6.1, in a direct comparison with the observed distribution to compute the p-value, the number of pseudoexperiments limits the possible maximal significance. Therefore the tail of the distribution of relevance values is again substituted by a fit following the decay of the tail of a Poisson distribution. The result can be seen in Figure 54. Just like for the fit of the plain relevance values in Figure 53, the actual histogram is used up to the threshold shown in black. The fit is only used for values above this threshold and hence does not at all influence any of the significances quoted in this thesis. The strong fluctuations in the left of the histogram in red aren't statistical fluctuations but systematic binning and combination effects. They are caused by the combination of the, in this case two, independent distributions from the two used segmentations, since for every entry in the histogram, the maximum renormalized relevance value found for this pseudo-experiment is used.

Finally, the distribution with the fitted tail from Figure 54 is used to compute a p-value for a found cluster. It is a one-sided p-value, since only overfluctuations are considered in this search. The same method could also search for

¹⁴ Using the boost library, see [104]. A scaling factor of $\sqrt{2}$ has to be applied to the results of this function to obtain the correct results.



Figure 54: Fit to the distribution of renormalized relevance values used for the computation of significances.

underfluctuations if several maximum operations were substituted by minimum operations. The obtained p-value is converted to a significance for this cluster using the inverse error function from [104] again. The resulting significance values correspond to the standard way of converting p-values to significances in Astroparticle physics, assuming a Gaussian distribution and a two-tailed evaluation.

8.7 Sensitivity

In this chapter the sensitivity of this method is illustrated for various source scenarios. Of course by design not all possible source morphologies that can be detected by the method can be covered, but instead several basic setups are shown to give an idea of the performance. Two relevance metrics, the size of the cluster N (Metric 0) and the average of the highest \sqrt{N} values in the cluster (Metric 1), are used for these plots. The sensitivities are calculated for pseudo-experiments with 13000 random background events with the quoted number of source neutrinos artificially added. Each of the spherical grids in this chapter is rotated such that it presents a reasonable view on the current source(s). The sensitivity curves are smoothed, since only 20 repetitions per data point could be conducted due to time constraints. The visualizations of the setups are shown with the maximum number of events per source tested in the corresponding sensitivity plots. To clarify explicitly, in this chapter the given sizes of the benchmarked sources are their radius, not their diameter.

8.7.1 Small and pointlike sources

Since dedicated, model optimized searches didn't find clear indications for pointlike sources, it is not to be expected for this search either. Figure 55 shows a setup with one almost pointlike source with a radius of 0.5° at a declination of -70° . The corresponding sensitivity curve in Figure 56 shows, how likely it is to detect the point source with a significance of 3σ , depending on the number of events randomly distributed within the source. This means that a point source of this size and location needs 12 events to have a chance of 50% to be detected with a significance of at least 3σ . Figures 57 and 58 show the corresponding information for a source with a radius of 1.0° .



Figure 55: Point source smaller 0.5° at a declination of $-70^\circ.$



Figure 56: Sensitivity for a point source smaller 0.5° at a declination of -70° .



Figure 57: Point source smaller 1° at a declination of $-29^\circ.$



Figure 58: Sensitivity for a point source smaller 1° at a declination of -29° .

8.7.2 Extended sources

The extended sources here are added using random positions for the events within the source. These tests show the performance for scenarios with extended sources like the Fermi Bubbles or the hotspot as seen by IceCube in [105]. The declination of the Galactic Center is at -29° , the hotspot at the time this is written is believed to be centered at -23° with a possible extension between 8° and 20° . Similar extended setups are shown in Figures 59 and 61. Analog to the point sources, the probability for a 3σ effect depending on the number of events within the source for these setups is shown in Figures 60 and 62. To give an impression how the result of the developed analysis looks like for extended sources, Figure 63 shows the result of the method for this scenario. This plot can be compared to Figure 50, which shows the same stage of the computation for the example with two point sources. Figure 64 shows the effect of a segmentation with $\alpha = -0.11$ on Figure 63. The corresponding plot for the two pointlike sources is Figure 52f. Every connected region here is one cluster. One can see that filaments are extending the cluster. This happens at locations where random accumulations of background events are close to the actual structure. The individual significance of each cluster is evaluated according to the chosen metrics. The cluster containing the artificial source events is detected with more than 5σ here. All other clusters on this sphere do not reach significances above 0.5σ .



Figure 59: Extended source smaller 5° at a declination of $+10^{\circ}$.



Figure 60: Sensitivity for an extended source smaller 5° at a declination of $+10^\circ.$



Figure 61: Extended source smaller 10° at a declination of -29° .



Figure 62: Sensitivity for an extended source smaller 10° at a declination of $-29^\circ.$

8.7 Sensitivity



Figure 63: Result for a setup like in Figure 61. This is after the summation of all search scales described in chapter 8.4, but before any segmentation cut is applied as described in chapter 8.5. The color scale can be considered arbitrary units, red means higher event density.



Figure 64: The effect of a segmentation with $\alpha = -0.11$ on the result shown in Figure 61.



Figure 65: The effect of a segmentation with $\alpha = +0.25$ on the result shown in Figure 61.

8.7.3 Unexpected shapes

A common idea of a source is a structure with an approximately spherical shape. This search does not require such a shape to find a source. The artificial setup of a straight line in Figure 66 is used to demonstrate that arbitrary shapes can be found. The sensitivity for this scenario is shown in Figure 67. In the scenario shown in Figure 68, the even more arbitrary shape of a line with a gap is shown. Whether this shape is reconstructed as one connected or two separate lines depends strongly on the number of events in the line and random fluctuations in the gap. The sensitivity for this setup, independent of whether a connected or two separate sources are identified, is shown in Figure 69. Figure 70 shows the detailed result obtained for this scenario before segmentation. One has to keep in mind, that this is for a scenario with unrealistically many events within the source. This result after the segmentation with $\alpha = -0.11$ is shown in Figure 71. The two parts have been detected as one long line in this example. Just like for the results for the extended sources, these results can be compared to the corresponding ones shown for the scenario with two point sources in Figure 50 and Figure 52f.

To demonstrate that the ability to identify arbitrary shapes is not limited to spherical and straight shapes, the setup shown in Figure 72 contains a curved line. The corresponding sensitivity can be seen in Figure 73. The results are shown in Figure 74 without segmentation and in Figure 74 after the segmentation using $\alpha = -0.11$. The general shape follows the desired directions, but there are considerable filaments extending beyond the region where the artificial events have been added. These are caused by random fluctuations near the actual source region, but there is no way to distinguish them from "real source events" in a real application.



Figure 66: Diagonal line, 20° length, at a declination of -70° .



Figure 67: Sensitivity for a diagonal line, 20° length, at a declination of -70° .



Figure 68: Diagonal line with interruption, 20° length, at a declination of -29° .



Figure 69: Sensitivity for a diagonal line with interruption, 20° length, at a declination of -29° .



Figure 70: Result for the setup in Figure 68. This is after the summation of all search scales described in chapter 8.4, but before any segmentation cut is applied as described in chapter 8.5. The color scale can be considered arbitrary units, red means higher event density.



Figure 71: The effect of a segmentation with $\alpha = -0.11$ on the result shown in Figure 68. The two parts of the line are connected in this example, but this is not the case for all pseudo-experiments of this setup, especially with less events.



Figure 72: Curved line, 20° length, at a declination of -29° .



Figure 73: Sensitivity for a curved line, 20° length, at a declination of -29° .



Figure 74: Result for the setup in Figure 72. This is after the summation of all search scales described in chapter 8.4, but before any segmentation cut is applied as described in chapter 8.5. The color scale can be considered arbitrary units, red means higher event density.



Figure 75: The effect of a segmentation with $\alpha = -0.11$ on the result shown in Figure 72. The filaments in this example are caused by random background fluctuations near the source.

8.7.4 Neighboring sources

If other sources are located close to a source, the probability to detect the presence of the whole structure is higher than the sum of the probabilities to detect the sources independently. This is demonstrated by the setup as seen in Figure 76. The sensitivity for this scenario is shown in Figure 77. The numbers of events are per source here. What can be seen is that the number of events required for each point source to obtain a 3σ effect with a certain probability is lower than for one source of the same extension and location alone. For comparison: The sensitivity for a single source of the same size and location has been presented in Figure 58.

8.7.5 Diffuse flux

This search is not at all sensitive to a completely diffuse flux. It adapts to the data in the beginning of the search in such a way that the overall excess is automatically compensated, making it completely impossible to detect anything that doesn't have a spatial structure.



Figure 76: Three point sources smaller 1° at declinations of -25° , -29° and -34° .



Figure 77: Sensitivity for three point sources smaller 1° at declinations of $-25^\circ,$ -29° and $-34^\circ.$
8.8 Modifications for IceCube data

8.8.1 Necessary modifications

Although no explicit assumption has been made derived from properties of the ANTARES neutrino telescope for this search, this happened implicitly at some points when the algorithm was optimized with the specific application in mind. If the same algorithm is to be applied to the public IC40 dataset [106], released by the IceCube collaboration, some details should be reconsidered. What was realized to be the relevant difference between the application to ANTARES and IceCube data is the approximation of the expectation of the number of neutrinos for a certain declination as described in chapter 8.3.3. While the probability to detect a neutrino changes slowly with the declination for ANTARES, see Figure 41, there is a leap for IceCube at a declination of about 0° as seen in Figure 78. The consequence of this discrepancy is that the estimated expectation derived from scrambled data cannot be smoothed as much for IceCube as for ANTARES, because otherwise the estimated expectation would strongly smear the observed jump. The number of applied lowpass filter operations is therefore reduced from 60 to 15 to better preserve this leap. The result of the acceptance estimation for the IC40 data can be seen in blue in Figure 78 for up to 0.5° and in Figure 79 for 10° .



Figure 78: Approximation of the acceptance of the IceCube detector and the reconstruction chain for a search distance up to 0.5° , based on the IC40 data sample.

Apart from this adaption, no part of the algorithm needs to be modified to



Figure 79: Approximation of the acceptance of the IceCube detector and the reconstruction chain for a search distance between 10.0° and 10.5° , based on the IC40 data sample.

apply it to the public IC40 dataset.

8.8.2 Optional modifications

Since the ANTARES and IceCube datasets are completely independent from each other but both should contain the same physical observation of the neutrino sky, one can be used to check a hypothesis derived from the other in the region where both datasets overlap without a trial factor.

A simple way to achieve this with this search method would be to analyze the IceCube data with the hypothesis generated by the ANTARES data and compare the results by eye. Since the outcome is not trivial to interpret, this approach could easily be misleading. To perform this check on a more solid basis, the idea is to predefine some requirements for the comparison of the hypothesis from ANTARES data and potential clusters in the IceCube data. First of all, the same segmentation threshold is used to evaluate the IC40 data as was used to find the most significant structure in the ANTARES data. Furthermore, a cluster has to overlap with the template of the ANTARES structure. Since large structures, which barely overlap with the template by only one pixel, certainly are not the aim of such a search, a minimum overlap is required for a cluster to be taken into account. This overlap requirement is set to 51% of the size of the new cluster to require it to be more related to the template than to other regions. Since requiring an overlap with a template implicitly means searching in a confined region, the metric to evaluate the significance cannot be size, as it would prefer clusters which exactly have a fraction of 51% of their size inside and 49% outside of the template. Therefore the metric is changed to metric number 1, topSqrt, as described in Appendix E. This metric uses the mean value of the \sqrt{N} highest pixel-values in the cluster, with N being the size of the cluster. Judging from pseudo-experiments, this is the second best tested metric.

Part III Results and discussion

This part of the thesis presents the results obtained using the described methods and gives an interpretation of these results.

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9 Results

Only after the development of the methods is finished and the analysis procedure and all cuts are fixed, the ANTARES data are processed for the first time, the so called "unblinding". This mandatory blinding policy prevents that a bias can influence the development of the methods or analyses. After filtering the recorded data as described in chapter 7, the unblinding for this analysis resulted in 13283 neutrino event candidates. The spatial distribution of these events has been analyzed using the techniques described in chapter 8. The result of this analysis are clusters, each with an independent significance. As explained in chapter 8.6.1, only the size of the clusters is used as relevance metric to assess their significance. Since two different segmentation thresholds have been used, there are two different resulting skymaps. The trial factors for all evaluations have been incorporated into the post-trial significance of each individual cluster.

9.1 Found clusters

The segmentation with a threshold of $\alpha = -0.11$, see chapter 8.5, is the harder of the two applied segmentation cuts. It detects multiple extended clusters which are close together. These clusters are not significant on their own. The result as computed on the search sphere can be seen in Figure 80. Figure 81 shows the same results as a skymap in equatorial coordinates, Figure 82 as skymap in galactic coordinates. All skymaps are presented in Hammer-Aitoff projection. The highest significance of 0.81σ is found for the cluster at the bottom, colored in red on the sphere and in white in the skymaps.



Figure 80: Result of the unblinding with a segmentation threshold of $\alpha = -0.11$ on the search sphere. Multiple extended clusters have been found, but no structure is significant on its own. a) View centered on the clusters. b) View of the opposite side.

The threshold of $\alpha = +0.25$ is the less strict segmentation cut. Its intention is to find very large structures or to allow neighboring structures to merge. The clusters found using $\alpha = -0.11$ fuse in this segmentation, resulting in one large



Figure 81: Result of the unblinding with a segmentation threshold of $\alpha = -0.11$ in equatorial coordinates. The colorcode is the significance in σ .

cluster with a post-trial significance of 2.85σ . Due to the compensation of a systematic effect investigated in chapter 9.7, this significance gets reduced to 2.52σ . The significance of the clusters observed for $\alpha = -0.11$ changes are only slightly reduced by this effect to 0.79σ . The very large structure can be seen on the search sphere in Figure 83. Figure 84 shows the result as skymap in equatorial coordinates, Figure 85 in galactic coordinates. This cluster encloses the Galactic Center, but it is larger than the largest extended structures that are currently suspected to emit neutrinos, for instance the Fermi Bubbles [94].

Since the obtained results aren't trivial to interpret, several further studies are presented in the following chapters.



Figure 82: Result of the unblinding with a segmentation threshold of $\alpha = -0.11$ in galactic coordinates.



Figure 83: Result of the unblinding with a segmentation threshold of $\alpha = +0.25$. A very large structure with a post-trial significance of 2.85σ is found. a) View centered on the cluster region. b) View of the opposite side.



Figure 84: Result of the unblinding with a segmentation threshold of $\alpha = +0.25$ in equatorial coordinates. The colorcode is the significance in σ .



Figure 85: Result of the unblinding with a segmentation threshold of $\alpha=+0.25$ in galactic coordinates.

9.2 Result before segmentation

Figures 86, 87 and 88 show the structures that have been found by the multiscale search after summation of the individual scales as described in chapter 8.4, but before the segmentations are applied as described in chapter 8.5. What can be seen is that the clusters are located where the highest overfluctations on all scales are observed, so this relation works as expected also for the final data analysis. Figures 89, 90 and 91 show the same plots, but this time with the position of the neutrino events as overlay. They allow to validate that these overfluctuation regions actually correspond to regions where many events are contained.

It has to be noted that the plotted variable is not directly the flux at that point, but it is influenced by all scales up to 90° around this point. One of the most relevant differences to a flux map is that the same cluster of events near other overfluctuations might stand out with a high value, while surrounded by underfluctuations, the same cluster can have a much lower, unremarkable value.



Figure 86: Result before the segmentation. a) View centered on the cluster region. b) View of the opposite side.



Figure 87: Skymap before the segmentation in equatorial coordinates.



Figure 88: Skymap before the segmentation in galactic coordinates.



Figure 89: Result before the segmentation with events in white. a) View centered on the cluster region. b) View of the opposite side.



Figure 90: Skymap before the segmentation in equatorial coordinates with the events in black.



Figure 91: Skymap before the segmentation in galactic coordinates with the events in black.

9.3 Comparison with results for scrambled data

To be able to compare the obtained results to skymaps without any sources, pseudo-experiments with randomly scrambled data have been conducted. As examples three randomly selected skymaps are presented here in Figures 92, 93 and 94 using exactly the same color code that has already been used for the results of the actual data analysis, for instance in Figure 87. They show that the individual overfluctuations observed in the recorded data are common, but their distribution across the sky is not.



Figure 92: Result for scrambled data set I in equatorial coordinates.



Figure 93: Result for scrambled data set II in equatorial coordinates.



Figure 94: Result for scrambled data set III in equatorial coordinates.

9.4 Subsampling

To check if the method requires exactly the 13283 selected events to give the observed result, tests with randomly subsampled datasets have been conducted. They were done with 11000 randomly chosen events out of the 13283, but also with a relatively small subsample of 5516 events to evaluate the potential behavior of this method on an event sample of the same size which has been used for the most recent ANTARES point source search, see [91]. This procedure has not been repeated more often due to time constraints (generating reasonable statistics for the pseudo-experiments takes most of the time) and also since the stability of the method has been analyzed in a different setup in chapter 9.5.

9.4.1 11000 events, random subsample I

A random subsampling of 11000 out of 13283 events is expected to find similar structures. Due to the nonlinearity of the used metric "Size" to assess the relevance of a cluster, the significance can be expected to vary considerably. The presented plots show the results in galactic coordinates.

The detailed structure resulting from this evaluation can be seen in Figure 95. The most significant cluster, found with a segmentation cut of +0.25, reaches 2.94σ and is shown in Figure 97. So in this case the random subselection even surpasses the significance of the full dataset. For the evaluation using a segmentation cut of -0.11, shown in Figure 96, the most significant cluster has 1.87σ , which also is higher than what was observed for the full dataset.



Figure 95: Result of random subsampling I of 11000 events before segmentation in galactic coordinates.



Figure 96: Result of random subsampling I of 11000 events with a segmentation threshold of $\alpha = -0.11$ in galactic coordinates.



Figure 97: Result of random subsampling I of 11000 events with a segmentation threshold of $\alpha = 0.25$ in galactic coordinates.

9.4.2 11000 events, random subsample II

A second random subselection of 11000 events shows a different picture. Although the skymap containing the detailed structure in Figure 98 looks similar to Figure 95, Figure 100 shows that not all overfluctuation regions merged for a segmentation threshold of $\alpha = 0.25$. The most significant cluster for this segmentation cut only reached 0.53σ . The most significant cluster in the evaluation using a segmentation cut of -0.11 shown in Figure 99 has 1.11σ , again more than what has been observed for this segmentation with the full dataset.



Figure 98: Result of random subsampling II of 11000 events before segmentation in galactic coordinates.



Figure 99: Result of random subsampling II of 11000 events with a segmentation threshold of $\alpha = -0.11$ in galactic coordinates.



Figure 100: Result of random subsampling II of 11000 events with a segmentation threshold of $\alpha = 0.25$ in galactic coordinates.

9.4.3 5516 events, random subsample I

Random subsamples of 5516 out of 13283 events can be expected to differ considerably more from the full data sample than samples with 11000 events. Also the significance of found clusters can be expected to vary even more. The motivation for the exact number of 5516 events is that this is the number of neutrino candidates used in the most recent ANTARES point source search.

The resulting detailed skymap is shown in Figure 101. The obtained significances are 0.81σ for $\alpha = 0.25$ and 0.60σ with $\alpha = -0.11$.



Figure 101: Result of random subsampling I of 5516 events before segmentation in galactic coordinates.

9.4.4 5516 events, random subsample II

A second random subsample with 5516 event has been processed. The resulting detailed skymap is shown in Figure 102. The obtained significances in this case were 0.10σ for $\alpha = 0.25$ and 0.07σ with $\alpha = 0.11$.

9.4.5 Result of random subsampling

Since most comparisons with subsampled datasets show a trend to obtain considerably less significant results with less events, the question arises if this is the normal effect of more statistics allowing a more precise measurement. Theoretically speaking, it could be the case that this is not due to statistical fluctuations, but that the method itself is not stable. If this were to be true, the outcome wouldn't stabilize even for a clearer source with more events.



Figure 102: Result of random subsampling II of 5516 events before segmentation in galactic coordinates.

9.5 Recreating the cluster in pseudo-experiments

This chapter shows the results of pseudo-experiments conducted to test if the outcome of the method stabilizes for strong sources. This helps to distinguish whether the method itself might be unstable for a structure like the observed cluster, or if the method stabilizes for strong sources and the observed behavior is best explained by the assumption that the structure is just at the edge of the detectable strength.

Pseudo-experiments have been simulated with a varying number of events M placed artificially within the template of the cluster. The location of the artificial events within the template is random, but according to the visibility of ANTARES. A random background of 13283 - M randomly scrambled events is added for each experiment to in total obtain the same number of events as in the actual data sample. This allows to generate a coarse curve of the obtained significance for varying number of events in the cluster, shown in Figure 103. For each of the data points 30 repetitions have been conducted.

Due to the strong nonlinearity of the method, the error bars are huge before the method stabilizes for clear signals. Unfortunately these large fluctuation in the transition region prevent an answer to where the observation might be located on that function. It has to be noted that the generated curve is not exact, since the events have been randomly distributed within the cluster. If they were to be distributed with an internal structure, less events would be needed to achieve the significances. But modeling these fluctuations can hardly be done without further speculative assumptions about the composition of the



Figure 103: Detected significance against the number of artificial events in the structure. The values are median values, the error bars represent 1σ ranges.

observed structures. Anyways it would only shift the stabilization to different numbers of events, not reduce the fluctuations themselves. Therefore the result of this check, that the method does give stable results for this cluster if it is strong enough, wouldn't change.

Another result of these tests is that the the cluster can be explained best by an excess between 200 and 300 events. Of course this test alone is no reliable estimate due to the large spread of the results around 200 added events. Another idea how to obtain the number of excess events in the cluster has been to estimate the number of events expected within the shape of the cluster. Pseudo-experiments yield a mean expected number of 1279 events, while in the measured data 1533 events are contained within the cluster shape. This would result in 254 excess events. Since these pseudo experiments do not account for the systematic effect presented in chapter 9.7, the actual number should be slightly lower. Based on all available information the best estimate for the number of excess events in the cluster shape is about 220 to 230.

9.6 Distributions

The intention of the checks of various distributions in this chapter is to either detect uncompensated systematic effects, which could have artificially created or fostered the observed cluster, or to notice potentially interesting deviations between the cluster region and the background. The first presented check of a distribution is the composition of the track reconstruction algorithms as selected by Selectfit seen in Figure 104. It shows how frequent each algorithm was selected for the events located within the cluster in red compared the events in the rest of the sky outside of the cluster in blue. A discrepancy between the two regions in this check could either reveal instabilities of Selectfit or show deviations in the properties of the events in and outside of the cluster. This could occur because the algorithms behave differently e.g. for events with different energies. Since not many of the events inside the cluster can be signal events even if there were to be very strong sources within the cluster, no deviation is to be expected.



Figure 104: Normalized fractions of the reconstruction algorithms selected by Selectfit in and outside of the cluster. 1=Aafit, 2=Bbfit, 3=Bbfit with M-Estimator, 4=Gridfit. No significant deviation between the regions is observed.

The next check in Figure 105 shows a comparison of the number of events in each month, accumulated over the years. What meets the eye is that the data taking efficiency drops considerably during spring and summer. This is mainly caused by increased bioluminescence reducing the number of clearly recognizable events. This effect is well known within the ANTARES collaboration and is not caused by any methods specific to this analysis. What can also be seen is that the trend is the same for the events within the cluster and outside of it, hence no additional systematic becomes obvious here.

Figure 106 shows a time resolved comparison too, but this time the number of events is plotted against the hour in local time at the ANTARES location (UTC+1, neglecting swapping between summer and winter time). While the



Figure 105: Comparison of the normalized fraction of events recorded in and outside of the cluster against the time of the year. There is a large seasonal variation observed in the ANTARES data taking, mainly due to different bioluminescence rates. No systematic deviations are obvious here between the cluster and the rest of the skymap.

number of events in the rest of the sky seems rather continuous, there are considerable deviations visible between the number of events detected within the cluster during day and night. There are multiple effects that can plausibly explain this behavior. First of all, obviously a different region of the sky is visible during the day than during the following night, simply due to the $\approx 180^{\circ}$ rotation of the Earth during the 12 hour period. If the data were taken during one single day and night only, the cluster with its maximal extension of up to 160° in right ascension (the core structure is about 40° wide) would be expected to be fully visible only for about 10.67 hours (2.67 hours for the core). These periods would be extended by a smooth transition of partial visibility for those times where parts of the cluster are not visible straight through the Earth, but with a different efficiency under a zenith angle between 180° (straight upgoing) and 90° (exactly horizontal).

Over the course of a year one would assume this visibility effect to be compensated by the rotation of the Earth around the Sun, because objects which are best visible during the daytime in summer are best visible during nighttime in winter. If one takes the observed effect of seasonal variations as seen in Figure 105 into account, it becomes clear that one expects a variation like the one seen in Figure 106 for a region of the sky which is best visible at night in the



Figure 106: Comparison of the normalized distribution of events in and outside of the cluster against the daytime the events were recorded in local time.

winter and at day in the summer.

As a matter of fact, the cluster is close to the region best visible during winter nights. The exact date and time with direct orientation on the cluster may not be perfectly clear from the Figures alone, but based on Figure 105, beginning of January seems like a reasonable guess, and based on Figure 106, the center of the overfluctuations could be around 1 am. On January 1st, at 1 am, above the opposite side of the Earth, the direct visibility of ANTARES is centered approximately at a right ascension of 287° and a declination of -19° . This is relatively close to the approximated center of the cluster region with a right ascension of 255° and declination of -15° , especially when the extensions of the cluster are considered.

A completely different effect, which could lead to a drop during daytime, is the operation of the telescope by humans. Reboots of unresponsive hardware are performed, experimental configuration setups are tested and calibrations of the detector are conducted, all during working hours, all introducing short periods of time where no useful data for a physics analysis can be taken. This explanation is also very plausible since the drop is observed between 8 am and 9 am, about the designated starting time of shift duties in ANTARES. It also ends between 17 pm and 18 pm, the reasonable estimate for the end of most shifts. If this effect is actually dominant, the drop should also be clearly visible in the distribution for all events against the local time, not just for the cluster. This comparison is shown as a zoom to the relevant range of the y-axis in Figure 109. Indeed, the drop is clearly visible, although not as pronounced as for the cluster.

Why is the drop not visible for the rest of the sky in Figure 106 then? From the first plausible explanation, the combination of efficiency variations throughout the year and differences in visibility, one would expect a weak overfluctuation for the rest during daytime. It cannot be as pronounced as the underfluctuation for the cluster, since the rest of the sky is bigger than the cluster region and therefore the variations in visibility must be less pronounced, but there should still be an overfluctuation there. Depending on the exact strength of the effect of human shifter interaction pulling in the opposite direction, anything between a slight overfluctuation (only the visibility effect contributes) and the same underfluctuation as observed for the cluster region (only the shifter interaction contributes) is compatible. The zoom to the relevant range of the y-axis for distribution of the rest of the sky alone in Figure 107 reveals, that in fact there is a small underfluctuation. Therefore the interaction of the shifters



Figure 107: Zoom to the normalized distribution of events outside of the cluster against the hour the events were recorded in local time.

seems to be the dominant effect.

It should be noted that other effects could possibly contribute to these variations, too. For instance a seasonal variation can be induced by the change of the effective temperature of the atmosphere on the other side of the Earth as described in [107]. It could possibly be noticed in a plot of the zenith angle of the events against the month. A seasonal effect on the number of neutrinos resulting from interactions in the atmosphere on the other side of the Earth

can be expected to move in zenith with the seasons. If on the other hand it is caused by seasonal variations at the ANTARES site, likely these wouldn't show a variation in zenith distribution. Unfortunately there is not enough statistics to answer this without a doubt.

To investigate a completely different topic, the number of events for each hour of data taking is presented in Figure 108. This is meant to check if a large number of events are recorded almost at the same time, for instance due to an unknown malfunction of the detector or a extremely unlucky configuration of bioluminescent macroorganisms or other unprecedented phenomena. If many events are recorded during such a short time period with approximately the same hit pattern, in theory they might be reconstructed as many neutrino events from similar directions. As one can see, there is no such accumulation in time.



Figure 108: Number of events inside the cluster against the hour the events were recorded. The aim of this plot is to detect spikes which could for instance indicate problems with the data taking at a certain time. No significant spike is observed.

Several further checks that showed no unexpected deviations beyond what can be presented here have been conducted. Some other checks, for instance a detailed evaluation of the energy spectrum, would be interesting at this point. But studies of plain physical properties would introduce a bias and therefore a trial factor for follow-up analyses, which should include these physical properties in a detailed evaluation to maximize their discovery potential. Since such a trial factor reduces their potential for a significant result, no properties which are likely to contribute to the significance of such an analysis are evaluated here prematurely.

9.7 Systematic effect of variable data taking efficiency

The combination of the variations observed in the data taking efficiency over the course of a day as seen in Figures 106 and 109, and of a year, seen in Figures 105 and 110, can potentially introduce an asymmetry in the data taking efficiency even in equatorial coordinates.



Figure 109: The distribution of all observed events against the hour of the day.

Pseudo-experiments have been conducted which include unrealistically strong variations for these distributions, resulting in an increased data taking efficiency near the location of the observed cluster. The deviations used have been 100% reduced efficiency during April, May and June and $\approx 50\%$ reduced efficiency during day time. The effect in equatorial coordinates is shown in Figure 111. The center of the created overfluctuation is located precisely at the estimated maximum of this effect as given already in the explanation of Figure 106 in chapter 9.6. Without the systematic effect the distribution of events would be expected to be similar to Figure 117.

As a next step, pseudo-experiments have been designed to model the realistic, actually observed variations in data taking efficiency. The resulting distributions for 30 combined pseudo-experiments, shown in Figures 112 and 113, match the actual variations nicely.

The resulting skymaps of the 30 pseudo-experiments can be seen in Figures



Figure 110: The distribution of all observed events against $\frac{1}{12}$ th of the year (Time periods of fixed length give a more stable impression than months).

114 and 115. They show only a weak trend to create or foster an extended structure near the observed cluster. The most significant result of the 30, shown in Figure 116, even shows a result uncorrelated to the result observed for the ANTARES data.

To compare the realistic effect on the number of events, a combination of 100 pseudo-experiments resulted in the distribution of events as shown in Figure 117. In contrast to the unrealistically strong variations from Figure 111, the effect is not visible anymore.

To quantitatively study the dipole-like impact of the systematic effect on the number of events, a projection of the events to one dimension, right ascension, is performed. As seen in Figure 118, the effect of the systematic on the normalized number of events is on the order of $\pm 1\%$, while the statistical fluctuations for each individual pseudo-experiment with 13283 ± 115 events are on the order of $\pm 10\%$.

Nevertheless there is a systematic effect increasing the probability to observe a cluster at the location found in the ANTARES data. A solution to compensate for it is to randomly redistribute the exact event times when scrambling the data instead of generating new random times anywhere within the data taking period as performed in chapter 8.3.3. This preserves the variations observed throughout the year and the day in the pseudo-experiments, hence they include the systematic effect. With this change for the pseudo-experiments, the significance of the observed cluster described in chapter 9.1 drops to 2.52σ (from



Figure 111: The effect of unrealistically strong variations of the efficiency of the data taking over time on the expected number of events in equatorial coordinates. In this color code, red indicates most, blue least events expected at a location. Image credit: Dr. Thomas Eberl, ECAP, FAU Erlangen.

 2.85σ). This way of producing pseudo-experiments slightly underestimates the significance, since pairs and groups of exactly the same events are more likely to reoccur due to the limited number of possible times to assign. This underestimation is well below 0.1σ , since using pseudo-experiments with a scrambling of the exact times, but with a randomized minute value, does not suffer from such a degeneracy and yields 2.61σ .

There is another aspect that could theoretically influence the probability to observe the measured cluster in the pseudo-experiments. When the scrambling of exact event times is considered to be exact, one silently assumes that the probability to measure an event at a time t_1 with properties p_1 (zenith, azimuth, energy, noise level, ...) is exactly the same as to measure an event with p_1 at a different time t_2 . If for example the zenith distribution were to be very different for day and night, the random scrambling would have to take that into account as well, for instance by scrambling only between time periods with similar distributions. The method described in chapter 8 evaluates only declination and right ascension. Hence only the parameters time, zenith and azimuth, from which the equatorial coordinates are derived, can potentially influence the result.

Earlier in chapter 9.7 studies have shown the variabilities in the time distributions, which on their own are compensated by the scrambling of exact event times. Neglecting even higher order effects, a systematic discrepancy between day and night would be expected to be necessary in one of the remaining relevant properties, zenith or azimuth, to foster an artificial cluster beyond the already discussed effects. Figure 119 shows the zenith distribution of the events separated into one day (all events between 8 - 20) and one night (20 -8) dataset. A comparison of the azimuth distributions of these datasets can be seen in Figure 120.

Since both distributions do not show systematic deviations, it is assumed



Figure 112: The distribution of the number of simulated events against the hour of the day.

here, that no additional systematic effect relevant for this analysis is introduced by variations of the probabilities to measure event properties.



Figure 113: The distribution of the number of simulated events against $\frac{1}{12}$ th of the year.



Figure 114: First 16 resulting skymaps from 30 pseudo-experiments with realistic data taking efficiency variations. (Zoomable)



Figure 115: Remaining 14 resulting skymaps from 30 pseudo-experiments with realistic data taking efficiency variations. (Zoomable)



Figure 116: Most significant cluster (2.96σ) of 30 pseudo-experiments with realistic data taking efficiency variations. It isn't notably similar to the results obtained for the ANTARES data.



Figure 117: The effect of realistic time variations in data taking efficiency on the expected number of events in equatorial coordinates. In this color code, red indicates most, blue least events expected at a location. Image credit: Dr. Thomas Eberl, ECAP, FAU Erlangen.



Figure 118: Variation in the normalized number of events projected to right ascension for pseudo-experiments with realistic time variations. a) b) and c) The distribution for one pseudo-experiment each. The statistical fluctuations dominate. a) Shows the distribution for the most significant pseudo-experiment as shown in Figure 116. d) The distribution for 100 times the statistic of one pseudo-experiment. The statistical fluctuations are reduced, the smaller systematic effect becomes visible. Image credit: Dr. Thomas Eberl, ECAP, FAU Erlangen.



Figure 119: The normalized distribution of events for different zenith bins and separated into day and night datasets.


Figure 120: The normalized distribution of events for different azimuth bins and separated day and night datasets.

9.8 Results for public IceCube data

A crosscheck with the publicly available IC40 dataset [106], published by the IceCube collaboration, is performed to see if the cluster found in the ANTARES data is also present in an independent dataset. To use the found cluster as a signal hypothesis, the search had to be modified slightly as described in chapter 8.8. The template used as hypothesis in this evaluation is exactly the result as shown in Figure 84.

The two datasets are well suited for a comparison because they contain the same physical observable, muon-neutrinos. Also the achieved direction resolution of both datasets is not very far apart, with the reconstruction of ANTARES being slightly more accurate (here 0.44° instead of 0.7°, see [108]). Although it is not necessary, the two datasets also happen to contain a similar number of events (13283 versus 12877 events). The only aspect that differs in a way that could influence the results notably is the different energy composition. Features that are observed in lower energies should be more pronounced in the ANTARES data, while aspects that are more dominant for higher energies should be better visible in the IC40 data sample. Still, overall this dataset offers an excellent way to crosscheck the ANTARES results.

The obtained resulting cluster from this evaluation can be seen in Figure 121. It has a significance of 2.14σ . Since the evaluation requires an overlap between the template from the ANTARES data and a cluster found in the IC 40 data, only clusters at the correct location are considered in this evaluation and are visible in the skymap. The detailed structures before any segmentation is applied can be seen in Figure 122 in equatorial, in Figure 123 in galactic coordinates. While some features observed in this skymap agree nicely with the ANTARES result, see Figure 87 for comparison, others don't. Besides the slightly different sensitivities in energy for the two datasets, it is already well known, e.g. from chapters 9.4, 9.5 and 9.7, that statistical fluctuations play a relevant role for some of the features found in these small datasets. Therefore the agreements and discrepancies between the results neither unambiguously confirm nor reject the result from the ANTARES data set.



Figure 121: Result of the evaluation of the IC40 data searching with a segmentation threshold of $\alpha = 0.25$ for a cluster that overlaps with the hypothesis from ANTARES data as explained in chapter 8.8 in equatorial coordinates.



Figure 122: Result of the evaluation of the IC40 data before segmentation in equatorial coordinates.



Figure 123: Result of the evaluation of the IC40 data before segmentation in galactic coordinates.

10 Interpretation of the results

The number of events obtained in this thesis is considerably larger than in most ANTARES analyses that apply the default methods and cuts. This gain is partially achieved by the improved event selection and reconstruction, but also due to looser cuts. With looser cuts, there are also more misreconstructed atmospheric muons in the obtained data set. Since misreconstructed events are distributed throughout the sky, their moderate increase constitutes no issue for an analysis aiming to detect spatially resolved sources.

The increased number of events is beneficial for this analysis not just by the availability of more statistics and therefore more signal, but it also improves the process of estimating the various expectations. Since in this search all expectations are derived from data, their estimation becomes more accurate with more data too.

The structures found with the segmentation cut of $\alpha = -0.11$ as seen in Figure 81 are not particularly uncommon, as they are neither significant by their size, nor does the detailed structure at their particular locations, seen in Figure 87, look special in any way, e.g. like unexpectedly strong overfluctuations. On the other hand the obvious fact that the clusters themselves are spatially clustered is special and is also the reason why the very large structure forms in the evaluation using $\alpha = +0.25$ as seen in Figure 84.

The details of these results are not completely trivial to interpret correctly. The statement that can be made for the large cluster based on the significance of 2.52σ is "An anisotropy in the variable evaluated in the multiscale search between the region exactly defined by the cluster shape and the rest of the sky with a size as large or larger as the one observed in the ANTARES data is generated with a probability of not more than p = 1.2% by a random distribution of events that includes the data taking systematics of ANTARES." The corresponding statements are of course also true for the less significant clusters found with the segmentation threshold of $\alpha = -0.11$.

The multitude of checks for systematic differences between the cluster and the rest of the sky, and also between the obtained data sample and the expectations provides additional insight into the result. The checks presented in chapters between 9.3 and 9.6 neither show any problems with the search method nor give clear hints on how to interpret the results. While these comparisons of the cluster and the rest of the sky show no obvious deviations from the expected behavior, these detailed checks led to the awareness of the relevant systematic effect investigated in chapter 9.7, which had been unaccounted for. Although it adds only a minor contribution to the observed result, investigations like this still can add to the understanding of the ANTARES neutrino telescope.

As already explained in chapter 8, the goal for this search, like for most model-independent analyses in other experiments, has been to identify the most significant excess beyond the expectations. This has been fully achieved by this search.

Since, on their own, all obtained results are well below 3.0σ , the common convention for an indication in Astroparticle physics, they are best explained by a random fluctuation of the background.

Nevertheless a few theoretically possible speculations which involve an actual neutrino signal shall be mentioned briefly here. First of all, no single object is likely to generate a cluster of the observed size. Although this is not disfavored by the obtained results, a more plausible speculation than one very large extended source region would be that multiple faint sources of unknown positions and extensions are distributed within the cluster together with random fluctuations. In simulations of similar scenarios the usage of a less strict cut for the segmentation value α allows neighboring simulated sources to be detected as one large cluster of higher significance. In this case the most likely location for sources would be at the positions of highest local overfluctuations in Figure 87. But even if this actually were to be true for some of these regions, others are guaranteed to be fostered or even caused by random background fluctuations as seen for example when comparing to the results in Figure 93. Therefore the highest overfluctuations cannot directly be considered "possible source locations" based on the results of this search alone. Furthermore, in simulations with remotely realistic artificial sources and positive α values used for segmentation, the obtained shape is usually not the exact shape of the artificial source, seen for instance in Figures 52b or 65. Therefore it is highly unplausible to assume that the detected shape of the large cluster should be exact. Random fluctuations at the edge are certain to influence its extension.

Concerning objects that could be expected to have similarly large extensions in a similar direction than the obtained result, one interesting aspect is that the observed cluster roughly extends in the direction of the center of our own galaxy (and beyond), but by eye its detailed structure does not coincide well with "usual galactic suspects" for neutrino sources like the Fermi Bubbles, see Figures 124 or the Galactic Center itself. Figure 125 presents the magnetic



Figure 124: The Fermi Bubbles are one of the typical candidates for extended galactic sources. This skymap is in galactic coordinates. Image Credit: NASA/-DOE/Fermi LAT/D. Finkbeiner et al. Taken from [109].

substructure of the Fermi Bubbles, which shows more similar structures like the obtained ANTARES results, but it's still no good match. Detailed searches for enhanced neutrino emission from these extended regions have been performed e.g. in [110] and [111] without finding a significant excess.



Figure 125: The magnetic substructure of the inner Galaxy in galactic coordinates. Shown is an overlay of the 33 GHz and 23 GHz total polarized intensity from 5 years data of WMAP. Blue and red are the shapes of the Northern and Southern Fermi Bubbles, yellow and green are the so called "Northern arcs". Taken from [112].

Another structure that comes to mind for these extensions is the Galactic Halo, described for instance in [113]. This diffuse, approximately spherical cloud of gas and stars around our galactic disk could house a structure with the observed extensions. In this context it can be noticed that in the detailed structure of the results in Figure 88 there is also an overfluctuation visible roughly in the direction of the Small and Large Magellanic Cloud, some of the closest smaller galaxies, see Figure 24, but not in the direction of the Andromeda Galaxy, the nearest galaxy with similar extensions than the Milky Way. Apart from a direct correlation with matter in galaxies (e.g. many faint galactic sources), dark matter could also be a candidate for a neutrino production mechanism in such a scenario, see for example [114]. Various other structures with similarly large extensions and approximately in the same direction in the sky are known, see for instance [115]. But just as for the other already mentioned objects, by the results alone there is no clear evidence to support any of these speculations, which even emphasizes the need for a dedicated follow-up analysis.

An overlay of the ANTARES skymap with the events found in the high energy starting events analysis by IceCube, described in [26], with four years of data, see [116], is show in Figure 127. This overlay is meant as an illustration



Figure 126: The anisotropy in the neutrino flux from dark matter selfannihilation is expected to follow the line-of-sight integral of the Navarro-Frenk-White halo profile. The profile is shown here (dimensionless) for the Northern hemisphere in equatorial coordinates, but with a different way of plotting right ascension. Image taken from [114].

only, no correlation analysis between the two results has been performed. The energies of the neutrinos differ, as the IceCube sample contains neutrinos with much higher energies than the ANTARES analysis. The crosses mark shower events, the x-es mark the position of tracks. It is important to note that the error on the position of the shower events is large, typically above 10°, see [27]. While the locations of some accumulations seem to agree by eye, e.g. left of the Galactic Center or at the top right end of the cluster, others do not agree. Therefore no unambiguous lesson can be learned from this comparison either.

It is noteworthy that the evaluation of the completely independent IC40 dataset found a cluster at the location predicted by the hypothesis derived from ANTARES data. Since the observed cluster at this position is not extraordinarily pronounced, it still does not provide an unambiguous confirmation of the hypothesis. It would be a mistake to straightforwardly combine the two obtained significances, since the they do not have exactly the same shape. But the evaluation cannot require them to have exactly the same shape, since the contribution of random fluctuations to the result is too large. Looking at the detailed structure of the result in Figure 122, the adaption of the search strategy to the visibility of IceCube as described in chapter 8.8 seems to slightly overestimate fluctuations at the edge of the visibility near the horizon. Nevertheless this does not invalidate the obtained results in any way, as the exact same ef-



Figure 127: Overlay of preliminary 4 year IceCube HESE events with the skymap derived from ANTARES data.

fect is also contained in every pseudo-experiment, from which the significance is derived in the end. What contributes to the ambiguity of the results is that not all features in the two skymaps in Figures 87 and 122 agree well. While this is not a flaw in the method, but simply the influence of random fluctuations on small samples, it demonstrates that both of these results obtained from small statistics are not accurate enough to trust every detail.

A dedicated follow-up analysis based on the obtained results would be of great value to improve the discrimination between a statistical fluctuation and a potential indication of a faint signal. The main advantage of such an analysis would be that the location, size and shape of the source hypothesis are known instead of having to search for all possible locations and sizes at once. This decisively reduces the trial factors, which have been incorporated into this analysis implicitly by the free search in the pseudo-experiments. Using a large, independent dataset would be ideal to confirm or exclude the presence of an increased flux of neutrinos within the identified region.

Putting it all in a nutshell, the conducted analysis has fully fullfilled the given task, to identify the region with the most significant excess of recorded neutrinos, and the presented observations are worth further investigation.

11 Summary

This thesis describes three new methods designed for the data analysis of the ANTARES neutrino telescope, but versatile enough for many other applications. The first algorithm is a highly optimized scheme to deal with the signal selection required to identify the desired neutrino signal within a vast amount of data. It uses adapted cascades of Random Decision Forest classifiers to achieve this goal. This approach has been incorporated into the data selection to benefit the data analysis performed in this thesis. The second algorithm is an efficient combination of multiple independent, already available algorithms for the direction reconstruction of the observed muon-neutrinos. Internally this selection is again based on a Random Decision Forest classification. It clearly outperforms any individual reconstruction algorithm and the method in general is flexible enough to be used for different reconstruction tasks or even to combine different types of signal in one analysis. This approach has also been employed to enhance the data selection. The third explained algorithm constitutes the main focus of this work. It analyzes the spatial distribution of recorded neutrinos to detect anisotropies of arbitrary extension, shape and internal distribution at any position in the sky. Since the evaluation is based entirely on observed data, no assumptions derived from theories or Monte-Carlo simulations are necessary. Therefore it is more robust against deviations from expectations and can easily be applied to similar tasks.

The evaluation of the recorded ANTARES data using all three described approaches results in a large structure with a post-trial significance of 2.52σ . Multiple checks of the result neither revealed any flaws in the method, nor did they help much to reveal the origin of the observed cluster. On the other hand a systematic asymmetry in equatorial coordinates caused by the time variations of the data taking efficiency of ANTARES was identified and studied. But this systematic effect is already included in the quoted significance of the result. A crosscheck on an independent dataset from IceCube with this method also found a cluster at the location expected from ANTARES data. This cluster results in a significance of 2.14σ . A more detailed comparison shows that not all features seen in the ANTARES result match the ones in the IceCube result and vice versa, but this certainly doesn't invalidate them considering the influence of random fluctuations on data samples of their size.

The goal of this search, to detect the most significant excess of neutrinos in the obtained ANTARES data sample, has been achieved fully. To either identify the results as statistical fluctuations or to detect them significantly a follow-up analysis optimized to investigate the derived hypothesis is highly recommended.

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Part IV APPENDIX

A Usage of Selectfit

The following examples are python code that demonstrates how to use Selectfit. They do not show the full scripts, but just the additional parts for Selectfit. A full implementation of the training examples working with ANTARES run-by-run simulations, version 2.2, can be found in Seatray under antares-rdfclassify/resources/scripts/SelectFit/ExampleStepX.py.

A.1 Application of existing Selectfit

Already trained RDFs are available at the computing center in Lyon at /afs/in2p3.fr/throng/km3net/i3-tools/params/antares-rdfclassify/2014-12-11. The files are called bestReco_AA_BB_BBME_Grid_fit_r14956.rdf for the selection

which algorithm to use and errorEstimate_AA_BB_BBME_Grid_fit_r14956.rdf for the error estimation. Both are trained with the tracksOnly flag, meaning that only Aafit, Bbfit, Bbfit MEstimator and Gridfit are considered. Here is how to use Selectfit with pretrained RDFs:

```
# Import the classes needed for Selectfit
from icecube.rdfclassify import selectfit
# Load the library used for classification
load("librdfclassify")
# This extracts all features from the reconstructions results
\# So the results must be already in the frame at this point
tray.AddModule(selectfit.SelectFitFeatureExtractor,"extractFeat")(
        ("trackOnly", False)
# True means Aafit, Bbfit, BBfit MEst and Gridfit
        \# False also adds Dusj and Q Strategy
# Selecting which direction reconstruction to use
tray.AddModule("I3RDFClassify","selectFit")(
        ("RDFToLoad", "bestReco.rdf")
        # The path and filename of an already trained RDF
        # The RDF must be trained with the same number of features
        \# and the best reco output classes
        )
# Estimating the error class of the used reconstruction
tray.AddModule("I3RDFClassify","errorEstimate")(
        ("RDFToLoad", "errorEstimate.rdf")
        \# Same as above, but trained with
        \# the error estimate output classes
        )
```

```
\# Collect the information stored by the classifications \# and write an AntaresRecoParticle to the frame named "SelectFit".
```

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```
# Zenith, Azimuth and QualityParam are set accordingly.
tray.AddModule(selectfit.SelectFitWriter, "writeSelectFitToFrame")
```

A.2 Training anew

Again the following pieces of code are just the additional parts for Selectfit. Have a look at antares-rdfclassify/resources/scripts/SelectFit/ExampleStep1.py to ExampleStep5.py for a chain of scripts that perform the training.

The first step for a training is to extract features from Monte-Carlo simulations. The class numbers are by default automatically set to the best reconstruction algorithm by the module SelectFitFeatureExtractor:

from icecube.rdfclassify import selectfit

```
tray.AddModule(selectfit.SelectFitFeatureExtractor,"extractFeat1")(
    ("fileOutputName", "./featureFilesBestReko/SelectFit.dat"),
    # The location to write the feature files to.
    # All files for the same classification
    # should end up in the same folder
    ("trackOnly", True)
    # True means Aafit, Bbfit, BBfit MEst and Gridfit
    # False also adds Dusj and Q Strategy
    )
```

The second step uses the written .dat files containing the features to train a RDF which will then be able to select the best reconstruction result for unseen events. Potential complaints about missing .meta files can be safely ignored for this step.

The third step is to extract the features for the error estimation. To be able to estimate what the error of the selected reconstruction for an event is, it must perform the selection first. Therefore this step extracts the features, uses them and the trained bestReco.rdf to select a reconstruction, and then evaluates the angular error of the selected reconstruction according to section 6.5. It is **very important to use different events** than the ones used for the first feature extraction! Using the same events again will result in an unrealistic, perfect selection of the direction reconstruction and therefore change the error estimation.

```
from icecube.rdfclassify import selectfit
load("librdfclassify")
```

```
tray.AddModule(selectfit.SelectFitFeatureExtractor,"extractFeat2")(
    # No need for a feature file here
    # the features are used in the Seatray frame directly
    ("trackOnly", True)
    )
tray.AddModule("I3RDFClassify","selectFit")(
    ("DataName", "SelectFitFeatures"),
    # (optional): The name of the features in the frame
    ("RDFToLoad", "bestReko.rdf"),
    # The filename of the RDF trained in step2
    )
# The class number for the features is changed here
# from best reconstruction to angular error class
tray.AddModule(selectfit.SelectFitErrorEstimator,"resetClassErr")(
    ("fileOutputName","./featureFilesErrorEstimate/errEst.dat")
    )
```

The fourth step again uses the written .dat files for training. This time the class numbers are set such, that the resulting RDF will be able to estimate the angular error of the selected reconstruction for unseen events. Again, potential complaints about missing .meta files can be safely ignored for this step.

After these four steps the training is done. The two resulting RDFs for selection and error estimation can be used just as explained in section A.1.

B Feature list for Selectfit

The features are stored in files ending in ".dat". The format of these files is one line per event, each line starting with the class number followed by the feature numbers. The ordering is the one presented in this list. All numbers within a line are separated by one whitespace. All variables are taken from the Seatray physics frame. The features for Selectfit are:

- 1. Number of pulses from "CalibratedPulses"
- 2. Zenith of Aafit from "AafitFinalFit"
- 3. Azimuth of Aafit from "AafitFinalFit"
- 4. Lambda of Aafit from "AafitLambdaFinalFit"
- 5. Beta of Aafit from "AafitErrorEstimateFinalFit"
- 6. Zenith of Bbfit from "BBFitTrack"

- 7. Azimuth of Bbfit from "BBFitTrack"
- 8. BBFitBChi2 of Bbfit from "BBFitBChi2"
- 9. BBFitTChi2 of Bbfit from "BBFitTChi2"
- 10. Zenith of Bbfit MEstimator from "BBFitMEstTrack"
- 11. Azimuth of Bbfit MEstimator from "BBFitMEstTrack"
- 12. QualityParam of Bbfit MEstimator from "BBFitMEstTrack"
- 13. Chi2 of Bbfit MEstimator from "BBFitMEstTrack"
- 14. NumberOfHits of Bbfit MEstimator from "BBFitMEstTrack"
- 15. NumberOfUsedLines of Bbfit MEstimator from "BBFitMEstTrack"
- 16. Zenith of Gridfit from "GridFit_FinalFitResult"
- 17. Azimuth of Gridfit from "GridFit_FinalFitResult"
- 18. QualityParam of Gridfit from "GridFit_FinalFitResult"
- 19. Ratio of Gridfit from "GridFit_Ratio"
- 20. Ratio of Gridfit from "GridFit_Ratio_Precut"
- 21. XParameter of Gridfit from "GridFit_X_parameter"
- 22. Zenith of Dusj from "DusjShowerRecoFinalFit_FitResult"
- 23. Azimuth of Dusj from "DusjShowerRecoFinalFit_FitResult"
- 24. QualityParam of Dusj from "DusjShowerRecoFinalFit_FitResult"
- 25. CutValue of Dusj from "DusjShowerRecoCutValues"
- 26. ReducedLogLikelihood of Dusj from "DusjShowerRecoFinalFitReduced-LogLikelihood"
- 27. ReducedLogLikelihood of Dusj from "DusjShowerRecoVertexFitReduced-LogLikelihood"
- 28. ReducedChi2 of Dusj from "ShowerIdentifierReducedChiSquare"
- 29. ReducedChi2 of Dusj from "FitTimeResidualChiSquare"
- 30. Zenith of QStrategy from "QStrategyReconstructionResult"
- 31. Azimuth of QStrategy from "QStrategyReconstructionResult"
- 32. QualityParam of QStrategy from "QStrategyReconstructionResult"
- 33. Chi2 of QStrategy from "QStrategyReconstructionResult"

C Sequential execution of linear filters

Multiple application of linear filters gives the same results as the one time application of a larger linear filter. Applying a linear filter twice that operates on three gridpoints is analog to a filter that operates on five gridpoints.

] = twice ſ 1 21 $\frac{1}{2}$ 1*[1 2 2*[0 1 0 1 1*[0 4 6 4

The weights in front of the filters multiply.

D Segmentation example

Figure 128 shows that different segmentation thresholds allow to observe completely different results.

E List of relevance metrics

This is an incomplete list of the metrics tested to assess the relevance of a found cluster 15 :

Direct metrics:

- 0. The size N (number of gridpoints) of the cluster
- 1. The mean value of the highest \sqrt{N} values in the cluster, topSqrt
- 2. The L2 norm of all values in the cluster
- 3. The maximum value maxVal of any gridpoint of the cluster
- 4. The *average* value of all values in the cluster
- 5. The L1 norm of all values
- 6. The mean value of the highest up to 9 values in the cluster, top9
- 7. The mean value of the highest up to 5 values in the cluster, top5
- 8. The number of surface gridpoints of the cluster
- 9. The mean value of the highest 9 values in the cluster, adding 0 for smaller clusters
- 10. The mean value of the highest 5 values in the cluster, adding 0 for smaller clusters

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 $^{^{15}\}mathrm{The}$ numbering presented here matches the numbering in the source code.

- 11. The ratio of N to the number of surface gridpoints
- 12. The average distance between all gridpoints of the cluster
- 13. The maximum distance between two gridpoints of the cluster
- 14. The ratio of the maximum to the average distance

Linear combination meta-metrics:

- 21. $\sqrt{maxVal} \cdot top5 \cdot \sqrt{L2}$
- 22. $average \cdot \sqrt{L2}$
- 23. $topSqrt \cdot \sqrt{L2}$
- 24. $topSqrt \cdot \sqrt{L2} \cdot \sqrt{maxVal}$
- 25. $average \cdot \sqrt{L2} \cdot \sqrt{maxVal}$
- 26. $average \cdot \sqrt{L2} \cdot \sqrt{top5}$

More general meta metrics:

- 28. The sum of the two metrics that overfluctuated most
- 29. The sum of the three metrics that overfluctuated most
- 30. The sum of the half of the direct metrics that overfluctuated most
- 33. The sum of the overfluctuation of all direct metrics
- 34. The L2 norm of the overfluctuation of all direct metrics

F Conducting the multiscale search

This section explains how to actually perform the multiscale search. It assumes that the required prerequisites (pretrained RDFs, various scripts, ...) have been obtained from the svn. An example setup can be found in the file "multiscale-ExampleScripts.tar.gz".

F.1 Filtering the data

The first step is extract the desired events. If a list of the events in equatorial coordinates is already available in the correct file format, this step may be skipped. The input for this step is required to be one large .i3 (or .i3.gz) file containing the whole data. Hint: .i3 files can be concatenated simply by "cat file1.i3 file2.i3 ... > filesAll.i3". This doesn't work for .i3.gz files without uncompressing them first.

For now we assume that all data runs to analyze are contained in a file called "allData.i3". The script "applyAllCuts.py" can then be used to filter the desired events by executing: "./applyAllCuts.py -i allData.i3". Depending on the size of the input file this step may take days. The script performs the following tasks:

- Remove all runs that have been used as burn sample (the run numbers are hardcoded).
- Remove known sparking runs (hardcoded run numbers).
- (For technical reasons) Remove events that occur multiple times.
- Keep only events from runs that fullfill all desired quality criteria (hard-coded run numbers).
- Use the pretrained up/down RDFs from the folder ./RDFs/ to get rid of events classified as downgoing.
- Use the pretrained Selectfit RDFs from the folder ./RDFs/ to apply Selectfit to each remaining event.
- Keeps only events that have an upgoing direction reconstruction as result of Selectfit.
- Keeps only events that have a Selectfit error estimate class less than 5.

The output is a file called allData_processed.i3.

F.2 Obtaining expectations

The next step is to extract the events from the .i3 file to ASCII files and to estimate the expected number of events as described in chapter 8.3.3. This is done by executing the command: "./doSignalFirstAnalysis.py –benchmark -1 -i allData_processed.i3". This writes the files "candidateDump.txt" and "candidateDumpRaw.txt", and the file

"normalizationDataStep1/normalizationResults_Size165016_bins181_nr1.nres2". The folder "normalizationDataStep1" is assumed to be present, it is not created by the script. The file candidateDump.txt is a plain ASCII text file containing all events. Each line contains one event in the file, with the information stored in the following ordering, each entry separated by one space: Right ascension, declination, time of the event, energy of the event (currently not implemented), and two zero values (the same functions that read and write this information are used for other computations where these values have a meaning). The file candidateDumpRaw.txt contains the same events, but with azimuth and zenith instead of right ascension and declination, so in local instead of equatorial coordinates. The file normalizationResults_Size165016_bins181_nr1.nres2 contains the estimation how many events are expected at every location of the search sphere for all search scales, derived from the event sample and then smoothed by lowpass filters.

If an already available event sample is to be used where only equatorial coordinates are available, this step is a bit different. The file candidateDump.txt is required to be present in the same format as described above. The only relevant values are right ascension and declination, all other values are not required in the current version and can be zero values. The file candidateDumpRaw.txt also needs to be present and must follow the described format since it is read in and parsed, but it may contain arbitrary information, e.g. it may be a copy of candidateDump.txt. The command to invoke the expectation estimation in this case is: "./doSignalFirstAnalysis.py -benchmark -10 -i allData_processed.i3". The scrambling in the pseudo-experiments to obtain all estimates is then not performed with random times when converting from local to equatorial coordinates, but random right ascensions are used instead. The input of a valid .i3 file is still required because the script expects this input, not to obtain any events. The result is stored in the file

"normalizationDataStep1/normalizationResults_Size165016_bins181_nr1.nres2" just as before. Inputs in galactic coordinates are not supported.

The difference in the number of lowpass filters required for IceCube data, explained in chapter 8.8.1, is to be changed in the source code before this step is executed. This adaption can be performed in the file "src/signal-first/private/signal-first/signalFirstInternal.cpp". To make this piece of code easier to find, the location for this task within the file is marked with the tag #CODETODETERMINETHENUMBEROFLOWPASS.

F.3 Conducting pseudo-experiments

To compute pseudo-experiments that are required for the determination of the renormalized relevances and the significances of clusters, the call is "./doSig-nalFirstAnalysis.py –benchmark -2 -i allData_processed.i3". The option -i all-Data_processed.i3 is only used for the first step, but again the current implementation complains if no valid i3 file is specified. By default the script is processing the whole i3 file now before doing the actual work. To save a large fraction of this time, the line "tray.Execute()" near the end of the script doSig-nalFirstAnalysis.py can now be replaced by "tray.Execute(10)". Of course this is optional. If you do this, don't forget to revert it to the original line before step 1 is executed again.

If no local coordinates are available (or they aren't to be used for whatever reason), the call changes to "./doSignalFirstAnalysis.py –benchmark -20 -i allData_processed.i3".

Both calls by default perform 20 pseudo-experiments. They store their results in the folder normalizationDataStep2 (the output fails if it doesn't exist). The files are called

clusters_segment(ALPHA_FOR_SEGMENTATION)_(UNIXTIME).txt, so for example clusters_segment0.250199999999964_1433713240.txt. Their format is one line per cluster, each line: Number of metrics, one value for each metric, number of gridpoints the cluster has, a list of the indices of all gridpoints of this cluster. Each value is separated by one space.

The segmentation thresholds to use can be changed near the beginning of file "src/signal-first/private/signal-first/signalFirstInternal.cpp" at the location marked with the tag #ALPHASEGMENTATIONSTOUSE. If a change of the used metrics is desired, this is also possible in file signalFirstInternal.cpp at the location tagged with #SETTHEMETRICSMASKHERE. Since all metrics are evaluated for every cluster in every evaluation, a mask is used to only keep the metrics that are desired. If the metric with index n (see list in Appendix E) is to be included in the evaluation, a line "metrics Mask $[\mathbf{n}]=1.0;$ " is to be added there.

Computing pseudo-experiments takes some time (about 15 minutes per pseudo-experiment with two segmentations on a Intel(R) Xeon(R) CPU X5650), but it can be parallelized easily in independent processes. To be able to perform various fits later on, a minimal number of pseudo-experiments is required. This has not been investigated in detail, but 800 experiments have been sufficient (to perform the analysis without error, not for statistical purposes).

The unixtime in the filename is used to distribute the output of many such processes computing simultaneously to independent result files in a simple way. With the computing power used for this thesis it only occurred in very rare cases that two processes wrote a result for the same computation during the same second and hence wrote uncoordinately to the same file. Such files can be identified easily, as they produce errors in the next step. If this has occurred, ALL file with this unixtime have to be deleted. If not ALL files with this unixtime in their filename are deleted, the next step will fail again, since it assumes, that the same number of pseudo-experiments are available for every segmentation and metric.

F.4 Renormalizing the relevances

As described in chapter 8.6, the renormalized relevances are not derived from even more pseudo-experiments, but instead are derived from the already available ones. This computation is done by calling "./doSignalFirstAnalysis.py –benchmark -4 -i allData_processed.i3". There is no difference anymore between evaluations starting with local or equatorial coordinates. The resulting output is also stored in folder "normalizationDataStep2" in files called hist_Size(ALPHA_FOR_SEGMENTATION)_Metric(NUMBER_METRIC).txt, e.g. hist_Size0.250199999999964_Metric4.txt. They contain the histograms of the relevance values for one segmentation and one metric. Additionally, the files

hist_Size(ALPHA_FOR_SEGMENTATION)_Metric(NUMBER_METRIC)fit.txt are stored, containing the histograms with the tail substituted by a fit. With this information this step can also compute the distribution of renormalized relevance values used later to determine the significance. This distribution is stored in folder "normalizationDataStep3" in file "normalizationStep3_Maxima.txt". For every pseudo-experiment it contains the maximum of the found renormalized relevance values. The folder has to exists already.

The savvy reader may have noticed that –benchmark -3 is missing. This step is deprecated. It can be used to greatly extend the number of evaluation metrics by the computation of various meta-metrics (Metrics based on other metrics). Since this didn't lead to an improvement of the final result, it has been dropped.

F.5 Evaluating the data

When all prerequisites are computed, finally the actual evaluation of the data sample can be performed. This is done by calling "./doSignalFirstAnalysis.py

-benchmark -5 -i allData_processed.i3". It evaluates the events from the file "candidatesDump.txt", uses the estimated expectation of event numbers from folder "normalizationDataStep1" to compute Poisson probabilities, uses the distributions of relevance values from pseudo-experiments from folder "normalizationDataStep2" to renormalize the relevances of found clusters, and finally it uses the distribution of the renormalized relevances from folder "normalizationDataStep3" to determine a significance for a found cluster. The results are various outputs in the vtk file format for some of the intermediate steps of the algorithm and the results of the evaluation for visualization with Paraview. The main results are also output as skymaps in Hammer-Aitoff projection labeled hammerProj*.txt in equatorial and galactic coordinates. They can be visualized e.g. by the following commands in gnuplot:

- set view map
- set palette defined
- splot "hammerProjEquatorial.txt" w p pt 7 ps 1 palette notitle

Note: If the window size of Gunplot and the point size are too small, nothing is displayed. In that case, enlarge the window.

Most steps also output some .txt files containing debug information on various computations which can also be plotted with Gnuplot.

F.6 Additional steps for specific tasks

Other values for benchmark are possible when calling doSignalFirstAnalysis.py. -benchmark 0 computes an evaluation for a simulated source scenario (Note: the - in front of e.g. -5 has been to indicate a negative number). For (positive) values from 1 up to 25, one value of a benchmark is computed as shown in chapter 8.7. The shown scenarios in the sensitivity chapter are examples of these 25. A value of -6 performs post-processing on previously computed sensitivity result files. The computations performed for -7, -80 and -9 assume that a file "./stencilGrid.grid" is present, which indicates a template where to look for a cluster. -7 reads this template and evaluates the sky normally, but sets all clusters that do not overlap with the template by at least 51% to 0.0σ . -80 computes pseudo-experiments that also perform this check for overlap with the template, hence incorporating the effect of the template into the results of the pseudo-experiments. These functions were necessary for the evaluation of the IC40 dataset. -9 allows to distribute additional events within the template. The number has to be hard coded in the source code. This is how the checks for Figure 103 have been done.

G Sources tested during development

The most common test setup used during the development of the multiscale search contained three sources of $3.0^{\circ} \times 3.0^{\circ}$ at a declination of -57° with 50° between them in right ascension with 6, 10 and 15 events, one extended source

of $10^{\circ} \ge 10^{\circ}$ with 60 events at a declination of 0° and one point source, $0.5^{\circ} \ge 0.5^{\circ}$, containing 6 events at -29° with a shift of 90° in right ascension with respect to the extended source.

Table 1 shows a list of the sources that have also been used to to test the method at various times during the development and therefore influenced the heuristic optimization that led to the choice of the metrics and segmentation thresholds. Apart from these, several variations of these setups have been tested, but they cannot all be listed here.

Source description	Extension	Declination	# Events
Point source	$0.5^{\circ} \ge 0.5^{\circ}$	-70°	0 - 20
Point source	$0.5^{\circ} \ge 0.5^{\circ}$	-29°	0 - 20
Point source	$0.5^{\circ} \ge 0.5^{\circ}$	+10°	0 - 20
Point source	$1.0^{\circ} \ge 1.0^{\circ}$	-70°	0 - 20
Point source	1.0° x 1.0°	-29°	0 - 20
Point source	1.0° x 1.0°	+10°	0 - 20
Extended source	$5^{\circ} \ge 5^{\circ}$	-70°	0 - 80
Extended source	$5^{\circ} \ge 5^{\circ}$	-29°	0 - 80
Extended source	$5^{\circ} \ge 5^{\circ}$	$+10^{\circ}$	0 - 80
Extended source	10° x 10°	-70°	0 - 150
Extended source	10° x 10°	-29°	0 - 150
Extended source	10° x 10°	$+10^{\circ}$	0 - 150
Extended source	$30^{\circ} \ge 30^{\circ}$	-29°	0 - 300
Extended source	$50^{\circ} \ge 50^{\circ}$	-29°	0 - 500
Diagonal line	$20^{\circ} \ge 0.5^{\circ}$	-70°	0 - 80
Diagonal line	$20^{\circ} \ge 0.5^{\circ}$	-29°	0 - 80
Diagonal line w. interruption	$20^{\circ} \ge 0.5^{\circ}$	-29°	0 - 80
Curved line	$20^{\circ} \ge 0.5^{\circ}$	-29°	0 - 80
Curved line w. interruption	$20^{\circ} \ge 0.5^{\circ}$	-29°	0 - 80
Three point sources	1.0° x 1.0°	-25°, -29°, -33°	0 - 20
Three point sources	1.0° x 1.0°	-66°, -70°, -74°	0 - 20
Three point sources	$1.0^{\circ} \ge 1.0^{\circ}$	-60°, -70°, -80°	0 - 20
Two extended sources	$5.0^{\circ} \ge 5.0^{\circ}$	-29° (20° apart in RA)	0 - 50
Two extended sources	$5.0^{\circ} \ge 5.0^{\circ}$	-70° (20° apart in RA)	0 - 50
Two touching diagonal lines	$20^{\circ} \ge 0.5^{\circ}$	-39°	0 - 50
Three extended sources	$3^{\circ} \ge 3^{\circ}$	-57° (50° apart in RA)	4 - 15

Table 1: Sources tested during development



Figure 128: Real world example of different segmentation thresholds on an almost randomly selected region north-east of Spain and west of Italy. Different thresholds (here: water depths) reveal different underlying structures, which are all valid. It is therefore not absolutely defined what parts should be regarded to be connected, multiple thresholds can be used to find the most interesting structures. Image Credit: Google Earth, Google.

H Used runs

Here are the run numbers of all data runs used for the analysis:

$\begin{array}{c} 25691 \ 25700 \ 25705 \ 25706 \ 25\\ 25800 \ 25863 \ 25864 \ 25872 \ 25\\ 25933 \ 25934 \ 25935 \ 25937 \ 25\\ 26987 \ 26121 \ 26121 \ 26122 \ 26\\ \end{array}$	5707 25713 25714 257155880 25892 25893 258955947 25953 25954 259555122 26124 26125	$\begin{array}{c} 25716 & 25717 & 25720 \\ 25905 & 25920 & 25921 \\ 25958 & 25964 & 25968 \\ 26161 & 26161 \\ \end{array}$	$25721 \ 25722 \ 25722 \ 25922 \ 25923 \ 25926 \ 25986 \ 25986 \ 25988 \ 26100 \ 26227 \ 26228 \ 26228 \ 26228 \ 26228 \ 26228 \ 26228 \ 2628$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$\begin{array}{c} 26271 & 26272 & 26273 & 26274 & 26\\ 26395 & 26396 & 26397 & 26398 & 26\\ 26534 & 26535 & 26536 & 26537 & 26\\ 26534 & 26535 & 26536 & 26537 & 26\\ \end{array}$	$\begin{array}{c} 6275 \\ 26276 \\ 26405 \\ 26406 \\ 26407 \\ 26407 \\ 26540 \\ 26542 \\ 26542 \\ 26542 \\ 26542 \\ 26543 \\ $	$\begin{array}{c} 26283 & 26331 & 26333 \\ 26463 & 26465 & 26469 \\ 26544 & 26545 & 26546 \\ 26544 & 26545 & 26546 \\ \end{array}$	26334 26335 2634026470 26471 2648026592 26593 26594	26345 26355 26361 26363 2636426483 26484 26485 26487 2652826606 26633 26637 26638 26649
268795 26797 26800 26871 26801 26801 26818 26818 26820 26821 26824 26 26870 26871 26872 26873	$\begin{array}{c} 6864 & 26702 & 26703 & 26704 \\ 6802 & 26803 & 26804 & 26805 \\ 6825 & 26826 & 26827 & 26828 \\ 6874 & 26875 & 26876 & 26877 \end{array}$	26806 26807 26808 26829 26830 26831 26878 26891 26893 26830 26833	226715 20716 20713 26810 26811 2681 26832 26833 2683 26894 26895 2689	226813 26814 26815 26816 26817 26723 26726 26817 26818 26816 26817 26868 26867 26868 26867 26868 26897 26898 26902 26903 26904
26905 26917 26918 26920 26 26974 26978 26982 26983 26 27094 27095 27100 27130 27 27185 27216 27217 27218 25	6936 26937 26938 26940 6984 26987 26994 26995 7143 27153 27162 27163 7226 27230	26956 26957 26958 27002 27003 27008 27164 27165 27170 27231 27232 27233	$ \begin{array}{c} 26959 \ 26963 \ 26964 \\ 27009 \ 27013 \ 27014 \\ 27172 \ 27173 \ 27174 \\ 27234 \ 27235 \ 2724 \\ \end{array} $	$\begin{array}{c} 26965 \ 26967 \ 26968 \ 26970 \ 26971 \ 27080 \ 27081 \ 27090 \ 27091 \ 27092 \ 27175 \ 27176 \ 27177 \ 27180 \ 27182 \ 27182 \ 27175 \ 27177 \ 27180 \ 27187 \ 27187 \ 27277 \$
$\begin{array}{c} 27290 & 27291 & 27292 & 27293 & 27\\ 27425 & 27426 & 27427 & 27440 & 27\\ 27550 & 27551 & 27558 & 27565 & 27\\ \end{array}$	7294 27296 27297 273517458 27460 27461 274627566 27567 27569 27571	$\begin{array}{c} 27352 \\ 27352 \\ 27463 \\ 27464 \\ 27572 \\ 27573 \\ 27573 \\ 27574 \\ 27573 \\ 27574 \\$	$\begin{array}{c} 27403 \\ 27403 \\ 27467 \\ 27467 \\ 27575 \\ 27576 \\ 27577 \\$	27406 27407 27408 27413 2742427471 27501 27502 27504 2750527578 27580 27581 27582 27616
$\begin{array}{c} 27617 \ \ 27618 \ \ 27620 \ \ 27622 \ \ 27\\ 27648 \ \ 27657 \ \ 27658 \ \ 27659 \ \ 27\\ 27746 \ \ 27748 \ \ 27748 \ \ 27751 \ \ 27\\ 27814 \ \ 27815 \ \ 27822 \ \ 27824 \ \ 27\\ \end{array}$	7623 27624 27625 27627 7660 27667 27668 27669 7752 27753 27754 27755 7752 27755 27825 27826 27828 27830	$27632 \ 27633 \ 27633 \ 27634 \ 27721 \ 27722 \ 27722 \ 27723 \ 27756 \ 27757 \ 27758 \ 27831 \ 27832 \ 27853 \ 2785$	$\begin{array}{c} 27635 \ 27638 \ 27640 \\ 3 \ 27724 \ 27726 \ 27727 \\ 3 \ 27760 \ 27761 \ 27765 \\ 3 \ 27871 \ 27872 \ 27874 \end{array}$	22764127642276442764627647 22772827742277432774427745 22776327764278082781227813 42787527887278902789127892
$\begin{array}{c} 27893 \ 27894 \ 27895 \ 27896 \ 27\\ 27990 \ 27991 \ 28007 \ 28034 \ 28\\ 28110 \ 28114 \ 28131 \ 28132 \ 28\\ 28244 \ 28246 \ 28\\ 282$	7897 27898 27900 279418035 28037 28044 280588133 28143 28148 2816128248 28261	27946 27952 27953 28061 28062 28063 28162 28163 28182	$ \begin{array}{c} 27954 \ 27963 \ 27979 \\ 28064 \ 28065 \ 28069 \\ 28211 \ 28224 \ 28221 \\ 28205 \ 28206 \ 28211 \\ \end{array} $	27980 27981 27982 27984 27988 28067 28068 28070 28071 28106 28226 28233 28235 28236 28237 28215 28216 28217 28236 28237
$\begin{array}{c} 28335 \ 28245 \ 28245 \ 28337 \ 28340 \ 28 \ 28395 \ 28401 \ 28402 \ 28404 \ 28 \ 28455 \ 28455 \ 28456 \ 28 \ 28456 \ 28 \ 28456 \ 28 \ 28456 \ 28 \ 28 \ 28 \ 28 \ 28 \ 28 \ 28 \ 2$	8341 28342 28343 28344 8421 28422 28343 28344 8421 28422 28432 28433 8457 28458 28459 28460	$\begin{array}{c} 28285 \\ 28345 \\ 28345 \\ 28434 \\ 28434 \\ 28441 \\ 28441 \\ 28494 \\ 28494 \\ 28494 \\ 28495 \\ \end{array}$	28303 28300 2831 28348 28350 2835 228443 28444 2844 528496 28497 2850	28352 28354 28364 28387 2838 28352 28354 28386 28387 2838 28446 28448 28450 28451 28452 28507 28508 28510 28512 28542
$\begin{array}{c} 28552 & 28553 & 28554 & 28563 & 28\\ 28622 & 28623 & 28624 & 28625 & 28\\ 28655 & 28681 & 28686 & 28687 & 28\\ 28726 & 28727 & 28728 & 28730 & 28\\ \end{array}$	$egin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c} 28583 & 28584 & 28585 \\ 28642 & 28643 & 28644 \\ 28713 & 28714 & 28715 \\ 28735 & 28736 & 28735 \end{array}$	28586 28613 28613 28613 28645 28647 28645 28647 28645 28717 28718 28717 28718 28761 2876	$5 \ 28616 \ 28617 \ 28618 \ 28620 \ 28621 \ 28650 \ 28651 \ 28652 \ 28653 \ 28654 \ 28720 \ 28721 \ 28721 \ 28722 \ 28723 \ 28724 \ 28726 \ 28765 \ 28766 \ 28767 \ 287767 \ 28767 \ 287767 \ 2$
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I Bibliography

References

- K. G. Jansky. Radio Waves from Outside the Solar System. Nature, 132:66, 1933.
- [2] National Aeronautics and Goddard Space Flight Center Space Administration. Multiwavelength Milky Way. http://mwmw.gsfc.nasa.gov/ mmw_product.html, 2015. 2015-03-19.
- [3] Viktor Franz Hess. Über Beobachtungen der durchdringenden Strahlung bei sieben Freiballonfahrten. *Physikalische Zeitschrift*, 13:1084—-1091, 1912.
- [4] J. Abraham et al. [Pierre Auger Collaboration]. Properties and performance of the prototype instrument for the Pierre Auger Observatory. *Nuclear Instruments and Methods A*, 523:50–95, 2004.
- [5] T. K. Gaisser, T. Stanev, and S. Tilav. Cosmic Ray Energy Spectrum from Measurements of Air Showers. http://arxiv.org/abs/1303.3565v1, 2013.
- [6] Pavel A. Cherenkov. Visible emission of clean liquids by action of γ radiation. *Doklady Akademii Nauk SSSR*, 2:451, 1934.
- M. Ageron et al. ANTARES: The first undersea neutrino telescope. Nucl. Inst. and Meth. A, 656(1):11–38, 2011.
- [8] I. A. Belolaptikov et al. The BAIKAL underwater neutrino telescope: Design, performance, and first results. Astropart. Phys., 7(3):263–282, 1997.
- [9] T. DeYoung. Neutrino astronomy with IceCube. Mod. Phys. Lett. A, 24(20):1543-1557, 2009.
- [10] Wolfgang E. Pauli. Offener Brief an die Gruppe der Radioaktiven bei der Gauvereins-Tagung in Tübingen. 1930. Letter.
- [11] C. L. Cowan et al. Detection of the free neutrino: a confirmation. Science, 124(3212):103–104, 1956.
- [12] D. Fargion. Deflection of massive neutrinos by gravitational fields. Lettere al Nuovo Cimento 9, 31(2):49–52, 1981.
- [13] V. Gribov and B. Pontecorvo. Neutrino astronomy and lepton charge. *Physics Letters B*, 28:493–496, 1969.
- [14] Y. Fukuda et al. Evidence for oscillation of atmospheric neutrinos. *Phys. Rev. Lett.*, 81:1562–1567, 1998.

- [15] M.C. Gonzalez-Garcia and Y. Nir. Neutrino Masses and Mixing: Evidence and Implications. *Rev. Mod. Phys.*, 75:345–402, 2003.
- [16] M.C. Gonzalez-Garcia and Michele Maltoni. Phenomenology with Massive Neutrinos. *Phys. Rept.*, 460:1–129, 2008.
- [17] K.A. Olive at al. (Particle Data Group). REVIEW OF PARTICLE PHYSICS. Chin. Phys. C, 38:090001, 2014.
- [18] Julien Lesgourgues and Sergio Pastor. Neutrino mass from Cosmology. Adv. High Energy Phys., 2012:608515, 2012.
- [19] Julien Lesgourgues and Sergio Pastor. Neutrino cosmology and PLANCK. New Journal of Physics, 16:065002, 2014.
- [20] PBS NOVA, Fermilab, Office of Science, United States Department of Energy, and Particle Data Group. The standard model of elementary particles. http://en.wikipedia.org/wiki/File:Standard_Model_of_Elementary_Particles.svg, 2015. 2015-03-28.
- [21] R. Davis et al. Search for neutrinos from the Sun. Phys. Rev. Lett., 20:1205–1209, 1968.
- [22] C. Grupen. Astroparticle physics. Springer-Verlag Berlin Heidelberg New-York, 2005.
- [23] K. Hirata et al. Observation of a neutrino burst from the supernova SN1987A. Phys. Rev. Lett., 58:1490–1493, 1987.
- [24] R. M. Bionta et al. Observation of a neutrino burst in coincidence with supernova 1987A in the Large Magellanic Cloud. *Phys. Rev. Lett.*, 58:1494–1496, 1987.
- [25] E. N. Alexeyev et al. Detection of the neutrino signal from SN 1987A in the LMC using the INR Baksan underground scintillation telescope. *Phys. Lett. B*, 205(23).
- [26] M. G. Aartsen et al. [IceCube Collaboration]. Evidence for High-Energy Extraterrestrial Neutrinos at the IceCube Detector. *Science*, 342:1242856, 2013.
- [27] M. G. Aartsen et al. [IceCube Collaboration]. Observation of High-Energy Astrophysical Neutrinos in Three Years of IceCube Data. *Phys. Rev. Lett.*, 113:101101, 2014.
- [28] E. Fermi. On the Origin of the Cosmic Radiation. *Physical Review*, 75:1169–1174, 1949.
- [29] J. Hester, A. Loll, (NASA, and ESA). The crab nebula. http://upload. wikimedia.org/wikipedia/commons/0/00/Crab_Nebula.jpg, 2015. 2015-04-05.

- [30] M. Su, T. R. Slatyer, and D. P. Finkbeiner. Giant gamma-ray bubbles from Fermi-LAT: Active galactic nucleus activity or bipolar galactic wind? *Astrophys. J.*, 724(2):1044, 2010.
- [31] U.F. Katz and Ch. Spiering. High-Energy Neutrino Astrophysics: Status and Perspectives. Progress in Particle and Nuclear Physics, 67:651–704, 2012.
- [32] L. Nellen, K. Mannheim, and P.L. Biermann. Neutrino production through hadronic cascades in AGN accretion disks. *Phys. Rev. D*, 47:5270–5274, 1993.
- [33] Angel R. Lopez-Sanchez, Australian Astronomical Observatory, and Macquarie University. Centaurus A multiwavelength. http://oldweb.aao. gov.au/local/www/alopez/images/Cen_A_multiwavelength_7.jpg, 2015. 2015-04-12.
- [34] C.A. Meegan, G.J. Fishman, and R.B. Wilson et al. Spatial distribution of γ -ray bursts observed by BATSE. *Nature*, 355:143, 1992.
- [35] N. Gehrels and P. Mészáros. Gamma-Ray Bursts. Nature, 337:932, 2012.
- [36] Julia Schmid. Searches for High-Energy Neutrinos from Gamma-Ray Bursts with the Antares Neutrino Telescope. PhD thesis, Friedrich-Alexander-Universität Erlangen-Nürnberg.
- [37] J. F. Beacom et al. Measuring Flavor Ratios of High-Energy Astrophysical Neutrinos. *Phys. Rev. D*, 68:093005, 2003.
- [38] Adriaan Heijboer. Track Reconstruction and Point Source Searches with ANTARES. PhD thesis, Universiteit van Amsterdam.
- [39] Florian Folger. Search for a diffuse cosmic neutrino flux using shower events in the ANTARES neutrino telescope. PhD thesis, Friedrich-Alexander-Universität Erlangen-Nürnberg.
- [40] P. A. Cherenkov. Visible radiation produced by electrons moving in a medium with velocities exceeding that of light. *Phys. Rev.*, 52:378—379, 1937.
- [41] Camille Moirenc and CNRS. Optical module. http://antares.in2p3. fr/Gallery/CNRS/LineConstruction/116_Antares_Vincenttriplet. jpg, 2015. 2015-04-09.
- [42] Ronald Bruijn. The Antares Neutrino Telescope: Performance Studies and Analysis of First Data. PhD thesis, Universiteit van Amsterdam.
- [43] Francois Montanet. Depiction of ANTARES overview. http://antares. in2p3.fr/Gallery/3D/antares_pub.jpeg, 2015. 2015-04-09.
- [44] S. Adrian-Martinez et al. The positioning system of the ANTARES Neutrino Telescope. JINST, 7:T08002, 2012.

- [45] B.A.P. van Rens. The software trigger in ANTARES. Antares Internal Note ANTARES-SOFT-2004-001, 2004.
- [46] M. De Jong. The ANTARES Trigger Software. Antares Internal Note ANTARES-SOFT-2005-005, 2005.
- [47] John Carr, Stephanie Escoffier, and Dmitry Zaborov. Proposition for an alternative trigger based on the T3 cluster trigger. Antares Internal Note ANTARES-SOFT-2007-016, 2007.
- [48] S. Fukuda et al. (SuperKamiokande). The Super-Kamiokande detector. Nucl. Instrum. Meth. A, 501:418, 2003.
- [49] Ulrich F. Katz (for the KM3NeT Collaboration). The ORCA Option for KM3NeT, 2013. PoS(Neutel 2013)057.
- [50] Bettina Hartmann. Reconstruction of neutrino-induced hadronic and electromagnetic showers with the ANTARES experiment. PhD thesis, Friedrich-Alexander-Universität Erlangen-Nürnberg.
- [51] C. Tamburini, M. Canals, and X.D. de Madron et al. Deep-Sea Bioluminescence Blooms after Dense Water Formation at the Ocean Surface. *PloS one*, 8:e67523, 2013.
- [52] Nick Hobgood. Comb jellyfish. http://en.wikipedia.org/wiki/File: Combjelly.jpg, 2015. 2015-04-12.
- [53] Kevin Raskoff and oceanexplorer.noaa.gov. Arctic comb jellyfish. http://en.wikipedia.org/wiki/File:LightRefractsOf_comb-rows_ of_ctenophore_Mertensia_ovum.jpg, 2015. 2015-04-12.
- [54] The ANTARES Collaboration. A deep sea telescope for high energy neutrinos. http://arxiv.org/abs/astro-ph/9907432, 1999. 2015-03-19.
- [55] M. Honda, T. Kajita, K. Kasahara, S. Midorikawa, and T. Sanuki. Calculation of atmospheric neutrino flux using the interaction model calibrated with atmospheric muon data. *Phys. Rev. D*, 75:043006, 2007.
- [56] Morihiro Honda, Takaaki Kajita, Katsuaki Kasahara, and Shoichi Midorikawa. Improvement of low energy atmospheric neutrino flux calculation using the JAM nuclear interaction model. *Phys.Rev.D*, 83:123001, 2011.
- [57] Thomas Eberl and Claudio Kopper. The SeaTray software framework. Antares Internal Note ANTARES-SOFT-2009-013, 2009.
- [58] Claudio Kopper et al. A software framework for KM3NeT. In Proceedings of the ICRC 2009 conference, 2009.
- [59] D. J. L. Bailey. Genhen v5r1: Software Documentation. Antares Internal Note ANTARES-SOFT-2002-004, 2002.

- [60] A. L'Abbate et al. GENHEN v6: ANTARES neutrino generator extension to all neutrino flavors and inclusion of propagation through the earth. Antares Internal Note ANTARES-SOFT-2004-010, 2004.
- [61] Alexander Kappes and Uli Katz. On the Use of Event Weights for the Simulation of Atmospheric Muons in ANTARES. Antares Internal Note ANTARES-SOFT-2005-003, 2005.
- [62] A. Carminati, G. Margiotta and M. Spurio. MUPAGE user guide. Antares Internal Note ANTARES-SOFT-2007-004, 2007.
- [63] P. Antonioli, C. Ghetti, E. V. Korolkova, V. A. Kudryavtsev, and G. Sartorelli. A three-dimensional code for muon propagation through the rock: Music. Astropart. Phys., 7:357–368, 1997.
- [64] M. De Jong. The TriggerEfficiency program. Antares Internal Note ANTARES-SOFT-2009-001, 2009.
- [65] thinkastronomy.com. The Galactic Coordinate System. http://www. thinkastronomy.com/M13/Manual/common/galactic_coords.html, 2015. 2015-05-16.
- [66] Infrared Processing and Analysis Center/Caltech & University of Massachusetts. The galactic plane. http://upload.wikimedia.org/ wikipedia/commons/e/e4/Milky_Way_infrared.jpg, 2015. 2015-05-16.
- [67] Stefan Geißelsöder. Classification of events for the ANTARES neutrino detector. Master's thesis, Friedrich-Alexander-Universität Erlangen-Nürnberg.
- [68] Thomas Wittenberg. Algorithms for Pattern Recognition and their Realisation 2. Lecture Slides, Selective Undergraduate Course, Friedrich-Alexander-Universität Erlangen-Nürnberg, 2008.
- [69] Lutz Priese and Patrick Sturm. Introduction to the Color Structure Code and its Implementation. In International Conference on Image Processing, ICIP '04, (Volume:2), 2004. http://www.uni-koblenz.de/~lb/lb_ downloads/download/csc.pdf.
- [70] Richard O. Duda, Peter E. Hart, and David G. Stork. Pattern Classification. John Wiley & Sons, 1998.
- [71] Joachim Hornegger and Dietrich W. R. Paulus. Pattern Recognition of Images and Speech in C++. Verlag Vieweg, 1997.
- [72] J. Han, M. Kamber, and J. Pei. *Data Mining*. Elsevier & Morgan Kaufmann, Third edition, 2012.
- [73] Tin Kam Ho. The Random Subspace Method for Constructing Decision Forests. *IEEE Transactions on Pattern Analysis and Machine Intelli*gence, 20:832–844, 1998.

- [74] Vladimir Bystritsky and Sergey / The ALGLIB Project Bochkanov. AL-GLIB. http://www.alglib.net, 2011. 2011-11.
- [75] L. Breiman, J.H. Friedman, R.A. Olshen, and C.J. Stone. Classification and Regression Trees. Wadsworth, Belmont, CA, 1984.
- [76] J. R. Quinlan. Induction of Decision Trees. Mach. Learn. 1, 1 (Mar. 1986):81–106, 1986.
- [77] Laurent Hyafil and RL. Rivest. Constructing Optimal Binary Decision Trees is NP-complete. *Information Processing Letters*, 5:15–17, 1976.
- [78] Kaggle. http://www.kaggle.com, 2015. 2015-03-28.
- [79] Thomas Kittler. Erweiterung der Ereignisklassifizierung des ANTARES Neutrinoteleskops mittels Random Decision Forests. Bachelors thesis, Friedrich-Alexander-Universität Erlangen-Nürnberg.
- [80] J.A. Aguilar et al. A fast algorithm for muon track reconstruction and its application to the ANTARES neutrino Telescope. *Astroparticle Physics*, 34:652–662, 2011.
- [81] Hans Joachim Bremermann. Optimization through evolution and recombination. 1962. in Self-Organizing Systems, M. C. Yovits, G. T. Jacobi, and G. D. Goldstein (eds), Spartan Books, Washington D. C.
- [82] Raymond Chiong, editor. Nature-Inspired Algorithms for Optimisation. Springer, 2009.
- [83] Thomas Kittler. Abschlussbericht zur Optimierung der Kennzahlen für den Random Decision Forest. Project report, Friedrich-Alexander Universität Erlangen-Nürnberg, 2014.
- [84] Geoffrey E. Hinton, Simon Osindero, and Yee-Whye Teh. A fast learning algorithm for deep belief nets. *Neural Computation*, 18:1527–1554, 2006.
- [85] Alex Krizhevsky, Ilya Sutskever, and Geoffrey E. Hinton. ImageNet Classification with Deep Convolutional Neural Networks. Advances in Neural Information Processing Systems, 25:1097–1105, 2012.
- [86] Erwin Visser and Stefanie Wagner. Studies of the performance of different track reconstruction strategies using the FilteringFit track reconstruction. Antares Internal Note ANTARES-SOFT-2013-002, 2013.
- [87] Florian Folger. Diffuse flux analysis using the Dusj Shower Reconstruction. Antares Internal Note ANTARES-PHYS-2013-012, 2013.
- [88] Florian Folger. Updates on the Dusj diffuse flux analysis. Antares Internal Note ANTARES-PHYS-2013-014, 2013.
- [89] Q. Dorosti Hasankiadeh and H. Lohner. Search for Showers Induced by Diffuse Flux of Cosmic Neutrinos. Antares Internal Note ANTARES-PHYS-2012-011, 2012.

- [90] The ANTARES data quality working group. Data quality flags. http://antares.in2p3.fr/internal/dokuwiki/doku.php?id= quality_flags, 2015. 2015-05-16.
- [91] S. Adrian-Martinez et al. Search for cosmic neutrino point sources with four year data of the ANTARES telescope. Astrophysics J., 53:760, 2012.
- [92] S. Adrian-Martinez et al. Search for muon neutrinos from gamma-ray bursts with the ANTARES neutrino telescope using 2008 to 2011 data. A&A, A9:559, 2013.
- [93] S. Adrian-Martinez et al. Searches for Point-like and Extended Neutrino Sources Close to the Galactic Center Using the ANTARES Neutrino Telescope. *The Astrophysical Journal Letters*, L5:786, 2014.
- [94] S. Adrian-Martinez et al. A search for neutrino emission from the Fermi bubbles with the ANTARES telescope. Eur. Phys. J. C, 74:2701, 2014.
- [95] S. Adrian-Martinez et al. First results on dark matter annihilation in the Sun using the ANTARES neutrino telescope. JCAP11, 032, 2013.
- [96] P. Bagley et al (KM3NeT Coll.). Technical Design Report. ISBN: 978-90-6488-033-9, Available from www.km3net.org, 2010.
- [97] Kitware Inc. VTK User's Guide. Kiteware Inc., 2010.
- [98] Kitware Inc. The ParaView Guide (Full Color Version): A Parallel Visualization Application. Kiteware Inc., 2015.
- [99] Thomas Williams and Colin Kelley. gnuplot 4.6 An Interactive Plotting Program. http://www.gnuplot.info/docs_4.6/gnuplot.pdf, 2014. 2015-06-16.
- [100] Bernd Jähne. Digital Image Processing. Springer, 2005.
- [101] William K. Pratt. Digital Image Processing: PIKS Scientific Inside. John Wiley & Sons, 2007.
- [102] Hans W. Schüßler, G. Dehner, R. Rabenstein, and P. Steffen. Digitale Signalverarbeitung 2: Entwurf diskreter Systeme. Springer, 2009.
- [103] John G. Proakis and Dimitris K. Manolakis. Digital Signal Processing. Prentice Hall, 2006.
- [104] The boost community. Boost C++ libraries. http://www.boost.org, 2015. 2015-03-28.
- [105] M. G. Aartsen et al. (IceCube Collaboration). Observation of High-Energy Astrophysical Neutrinos in Three Years of IceCube Data. *Phys. Rev. Lett.*, 113:101101, 2014.

- [106] R. Abbasi et al. (IceCube Collaboration). A Search for a Diffuse Flux of Astrophysical Muon Neutrinos with the IceCube 40-String Detector. *Phys. Rev. D*, 84:082001, 2011. arXiv:1104.5187, http://icecube.wisc. edu/science/data/ic40.
- [107] S. Cecchini. Is there a seasonal temperature wave in the neutrino counting rate of ANTARES? ANTARES internal note ANTARES-PHYS-2011-007, 2011.
- [108] Albrecht Karle (for the IceCube Collaboration). IceCube. In Proceedings of the ICRC 2009 conference, 2009.
- [109] Trent Perrotto (NASA Headquarters) and Md.) Lynn Chandler (Goddard Space Flight Center, Greenbelt. NASA's Fermi Telescope Finds Giant Structure in our Galaxy. http://www.nasa.gov/mission_pages/GLAST/ news/new-structure.html, 2015. 2015-08-18.
- [110] Steffen Hallmann. Search for neutrinos from the Fermi Bubbles with the ANTARES telescope. In *Proceedings of the ICRC 2015 conference*, 2015.
- [111] Erwin Lourens Visser. *Neutrinos from the Milky Way*. PhD thesis, Universiteit Leiden.
- [112] David I. Jones, Roland M. Crocker, Wolfgang Reich, Jürgen Ott, and Felix A. Aharonian. Magnetic substructure in the Northern Fermi Bubble revealed by polarized microwave emission. *The Astrophysical Journal*, 747:L12 (4pp), 2012.
- [113] Paul Murdin. Encyclopedia of Astronomy and Astrophysics. Nature Publishing Group, 2001.
- [114] M. G. Aartsen et al. (IceCube Collaboration). Multipole analysis of Ice-Cube data to search for dark matter accumulated in the Galactic halo. *European Physical Journal C*, 75:20, 2015.
- [115] R. J. Egger and B. Aschenbach. Interaction of the Loop I supershell with the Local Hot Bubble. Astronomy and Astrophysics, 294(2):L25–L28, 1995.
- [116] Olga Botner. IceCube Present and Future. In *The IceCube Particle Astrophysics Symposium*, 2015. https://events.icecube.wisc.edu/conferenceTimeTable.py?confId=68#20150504.